A non-uniform node finite difference algorithm

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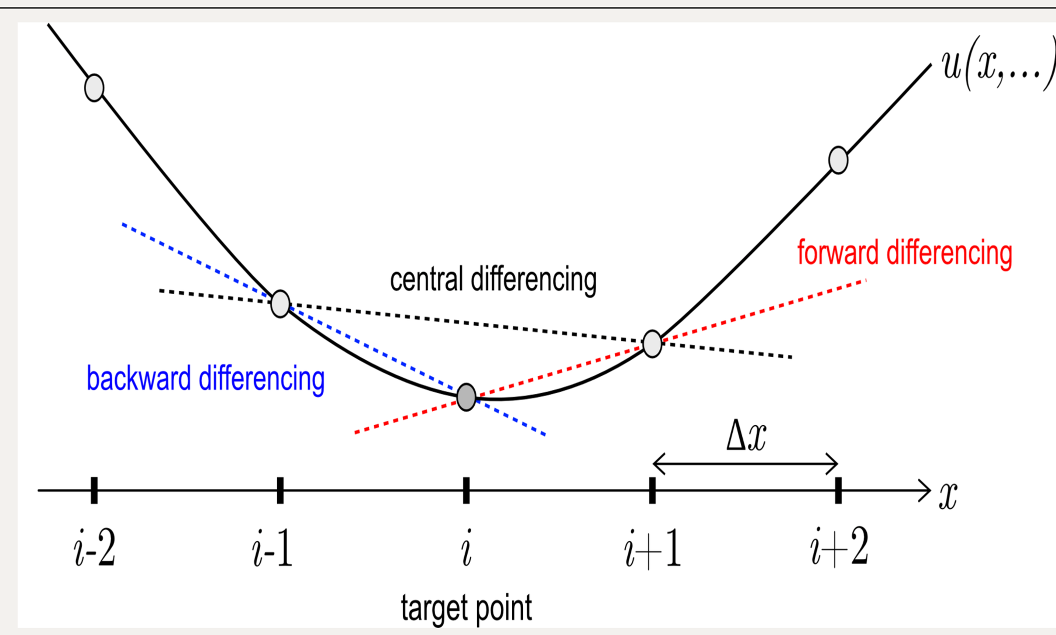
Abstract

This thesis primarily investigates the application of the **Finite Difference Method (FDM)** in solving Ordinary Differential Equations (ODEs). Initially, the paper meticulously derives the FDM algorithm for uniform nodes, utilizing the perspective of Taylor expansion. Following this, the paper elucidates how the uniform node FDM algorithm can be employed to transform the problem of marginally-valued ordinary differential equations into a system of linear equations. Subsequently, the paper introduces **non-uniform nodes** into the FDM algorithm, reconstructs the finite difference, and proposes an FDM algorithm for non-uniform nodes. Finally, the paper employs MATLAB to simulate both the uniform node FDM algorithm and the non-uniform node FDM algorithm. The **simulation results** demonstrate that the accuracy of the non-uniform node FDM algorithm is ten times greater than that of the uniform node FDM algorithm, and it can fit the actual ODE results well.

Introduction

In machine learning and deep learning, it is a common problem to find the derivative of a fixed node of a function, and computers need certain algorithms to complete this problem. FDM is one such algorithm. In this paper, a finite difference algorithm based on non-uniform nodes is proposed, and the accuracy of the finite difference algorithm for uniform nodes is improved ten times through numerical simulation

Principles of FDM Algorithms



Using the Taylor expansion, a finite number of points can be used to fit the derivatives at a fixed node, and the figure shows an example of a one-dimensional finite difference, for the intermediate points of the First order derivatives, using the finite difference algorithm, there are

$$u'(x_1) \approx \frac{-3u(x_1) + 4u(x_2) - u(x_3)}{2\Delta x} \tag{1}$$

This algorithm requires equal distances between nodes, which is relatively simple and easy to understand, but brings the disadvantage of making the algorithm's accuracy limited and not maximising the use of data.

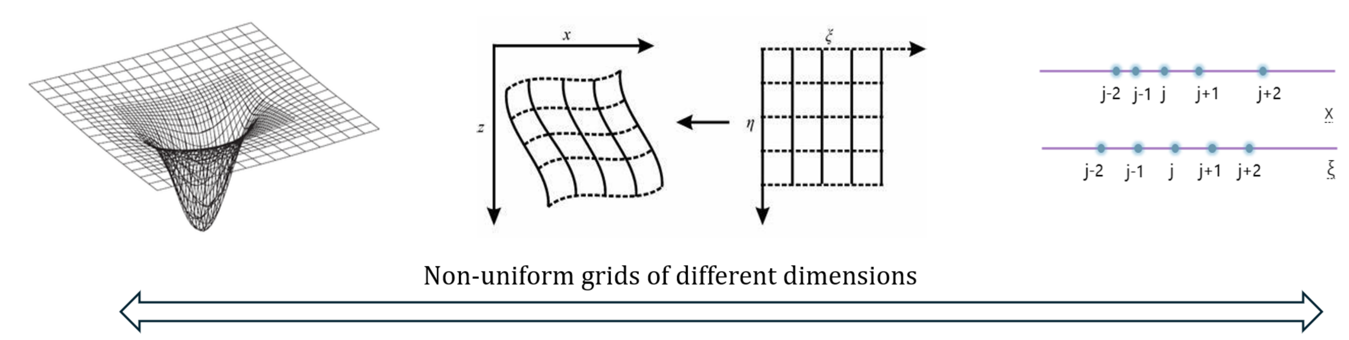
Methods

The article analyses the shortcomings of the existing algorithms and proposes a finite difference algorithm based on non-uniform nodes, which is studied as follows:

1. **Principles of FDM Algorithms**. In order to study how to better improve the accuracy of the uniform node finite difference algorithm, the In this paper, the principle of uniform node finite difference algorithm is carefully derived using Taylor's formula in preference.
2. **Introduction of finite difference matrices**. In the analysis of real problems, the amount of data are quite large, this paper, after introducing the uniform node finite difference algorithm principle, finite difference matrices are introduced to provide a method for quickly solving ordinary differential equations of edge-valued type.

$$\text{Coefficient matrix} = \begin{bmatrix} 1 & 0 & & \cdots & 0 \\ 1+0.5nh^2 & -2-nh^2 & 1-0.5n^2h & & \\ & \ddots & \ddots & \ddots & \\ & 1+0.5nh^2 & -2-nh^2 & 1-0.5n^2h & \\ 0 & \cdots & 0 & -1 & h+1 \end{bmatrix} \tag{2}$$

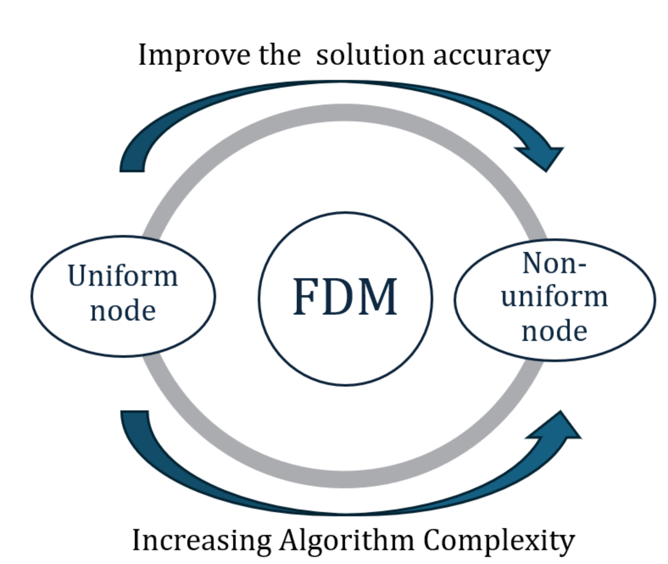
3. **Introducing the concept of non-uniform nodes**. By analysing that uniform nodes inevitably bring about a degradation of the solution accuracy. The article proposes to use non-uniform nodes as the nodes of the finite difference algorithm.



4. **Numerical simulation using MATLAB**.The feasibility of the proposed algorithm in the article is verified by numerical simulation through MATLAB.

Conclusions

Through numerical simulation, the accuracy of the finite difference algorithm based on non-uniform nodes proposed in this paper is **ten times higher** than that of the traditional uniform node finite difference algorithm. It can well cope with the problem of solving derivatives or differential equations in real life, but it brings the problem of **increasing computational complexity**, which may lead to the slowing down of the computation speed.



Results

In the article, a classical edge-valued ordinary differential equation problem is selected to be solved using uniform node finite difference solution and non-uniform node finite difference solution respectively for Numerical simulations were performed to obtain their solutions as follows

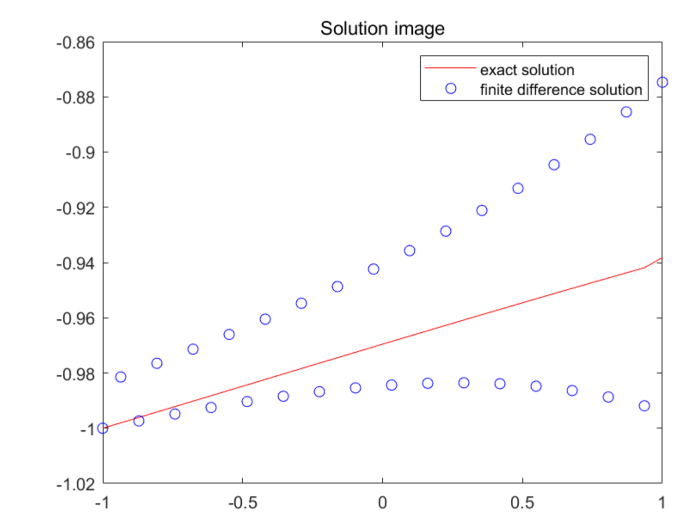


Figure 1. uniform node solutions

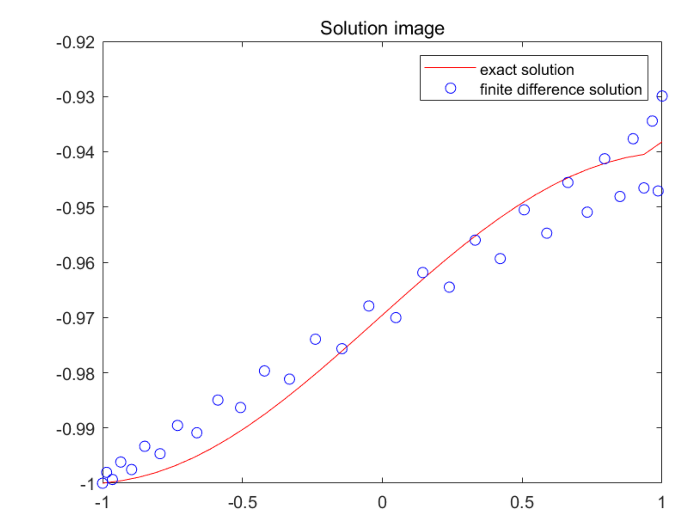


Figure 2. Non-uniform node solutions

And their relative and absolute error curves were obtained respectively as shown in Fig.

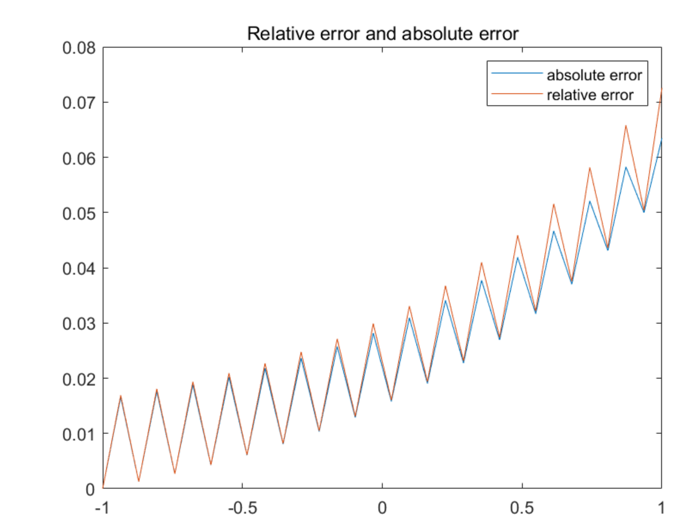


Figure 3. uniform node error

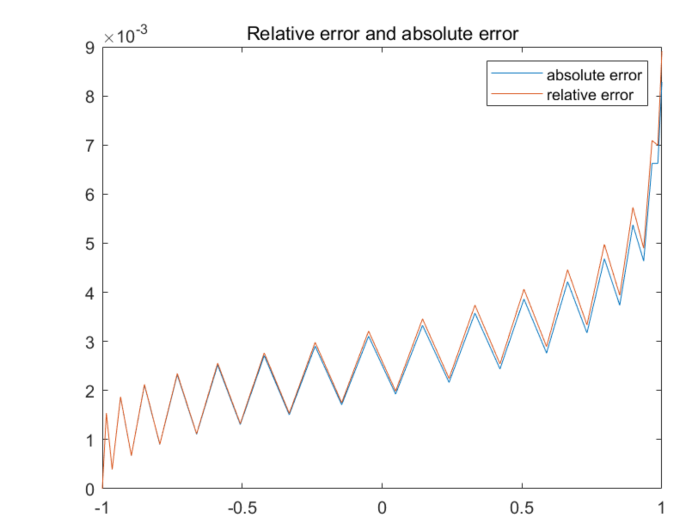


Figure 4. Non-uniform node error

Through the images, we can see that the error of finite difference of uniform nodes is not more than 10.0%, while the error of finite difference of non-uniform nodes is not more than 1.0%. This indicates that the algorithm proposed in this paper brings nearly **tenfold improvement in accuracy**.

References

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