复变函数

一、Complex Numbers and Complex Functions.

二、Analytic Functions.

三、Integral of Complex Function.

四、Complex Series.

五、Residues and Its Application

Analytic. 函数在point 的某些邻域上解析，该point 解析.

Singular. 函数在某点不解析，邻域包含所有邻域都有解析点.

isolated. 函数只在该点不解析，孤立奇点

一、Complex Numbers and Complex Functions.

$$\operatorname{Arg} z = \begin{cases} \arctan\frac{y}{x}, & x \ge 0 \\ \arctan\frac{y}{x} + \pi, & x<0, y\ge 0, \text{第二象限}. \\ \arctan\frac{y}{x} - \pi, & x\le 0, y<0, \text{第三象限}. \end{cases}$$

$\boxed{-\pi < \operatorname{Arg} z \le \pi}$

$z\cdot\bar{z} = |z|^2$.

$\arg(z_1 z_2) = \arg z_1 + \arg z_2$.

$\lim_{z\to z_0} f(z) = \lim_{z\to z_0} u(x,y) + i v(x,y) = \lim_{(x,y)\to(x_0,y_0)} u(x,y) + i v(x,y) = k$.

x,y 以任意曲线方式逼近 (x_0,y_0) 均得到 k, 极限存在

$\lim_{z\to\infty} f(z) = \infty \Leftrightarrow \lim_{z\to 0} \frac{1}{f(z)} = 0$. ，使用极坐标形式求极限較容易.

二、Analytic Functions.

1. 导数

$f(z) = u(x,y) + i v(x,y)$ 有条件 (C-R equation) $\begin{cases} u_x(x_0,y_0) = v_y(x_0,y_0). \\ u_y(x_0,y_0) = -v_x(x_0,y_0) \end{cases}$

那么 $f'(z_0) = u'(x,y) + i\, v'(x,y) = \underbrace{u_x(x_0,y_0) + i\, v_x(x_0,y_0)}$.

利用极坐标变换，若 $f(z) = u(r,\theta) + i v(r,\theta)$ 满足 $\begin{cases} v_\theta = r u_r \\ u_\theta = -r v_r \end{cases}$

则有 $f'(z_0) = e^{-i\theta_0}(u_r(r_0,\theta_0) + i v_r(r_0,\theta_0))$.

f在一点~~解~~析，~~只要在一点的某邻域解析~~ 邻域都解析.

f在某一点不~~解析~~ 但周围~~点都解析~~ 邻域有解析点，则自该为奇点.

孤立奇点，只在~~点处~~不解析.

2. Harmonic functions.

$H_{xx} + H_{yy} = 0$, H 就为调和.

如果 $u(x,y)$ 和 $v(x,y)$ 都调和并且满足 C-R equation，v是u的 $\boxed{\text{调和共轭}}$ ⇒ analytic.

3. Elementary functions.

$$\log z = \ln r + i(\theta + 2n\pi) \xRightarrow{\text{主值}} \operatorname{Log} z = \ln|z| + i \operatorname{Arg} z.$$

$\frac{d}{dz}\log z = \frac{1}{z}$，但是有一个 cut branch 不可导.

$$\sin z = \frac{e^{iz} - e^{-iz}}{2i}, \quad \cos z = \frac{e^{iz} + e^{-iz}}{2}.$$

$$\sinh z = \frac{e^{z} - e^{-z}}{2}, \quad \cosh z = \frac{e^{z} + e^{-z}}{2}.$$

反函数 $z = \sin w = \frac{e^{iw} - e^{-iw}}{2i} \Rightarrow w = \sin^{-1} z.$

三. Integral of Complex Function.

$$\int_a^b w(t)\,dt = \int_a^b u(t)\,dt + i\int_a^b v(t)\,dt = W(t)\Big|_a^b.$$

1、围线积分. $L = \int_a^b |z'(t)|\,dt = \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2}\,dt$ $(z = x(t) + iy(t))$.

$$\int_C f(z)\,dz = \int_a^b f[z(t)]\,z'(t)\,dt. \sim \text{计算非封闭路径}.$$

$$\int_C \frac{1}{(z-z_0)^{n+1}}\,dz = \begin{cases} 2\pi i, & n=0 \\ 0, & n \neq 0. \end{cases}$$

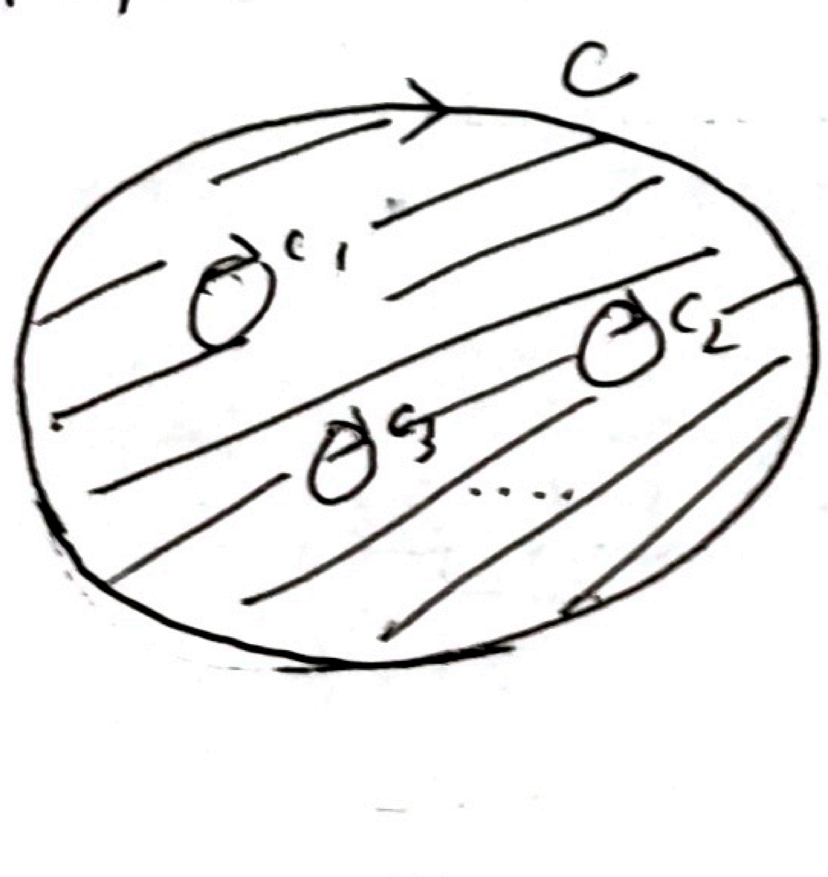
$\boxed{f(z)\text{有原函数} \Longleftrightarrow \text{路径无关} \Longleftrightarrow \text{闭环积分为}0.}$

2. Cauchy integral theorem.

① 使用 C-R equation 判断 $f(z)$ 在闭环围成内解析，则 $\int_C f(z)\,dz = 0$ (即域内解析).

② 解析域内路径无关.

③



$f(z)$ 在围线之中间隔处处解析，有

$$\int_C f(z)\,dz = \int_{C_1} + \int_{C_2} + \int_{C_3} f(z)\,dz.$$

3、Cauchy integral formula.

$$\int_c \frac{f(z)}{z-z_0} dz = 2\pi i f(z_0).$$ — 围线积分解只与奇点相关.

4、Higher-order Cauchy integral formula

$$\int_c \frac{f(z)}{(z-z_0)^{n+1}} dz = \frac{2\pi i}{n!} f^{(n)}(z_0)$$

III. Complex Series.

1、绝对收敛 → 收敛.

2、Power Series.

收敛域判断. $\lim\limits_{n\to\infty}\left|\frac{a_{n+1}z^{n+1}}{a_n z^n}\right| < 1 \Rightarrow |z|$的范围即收敛域.

或 $\lim\limits_{n\to\infty}\sqrt[n]{|a_n z^n|} < 1 \Rightarrow |z|$的范围即收敛域.

Abel's theorem. z_0收敛, $|z|<|z_0|$绝对收敛.

z_0收敛, $|z|>|z_0|$发散.

3、Tylor Series.

$$f(z) = \sum_{n=0}^{\infty} a_n (z-z_0)^n, \quad a_n = \frac{f^{(n)}(z_0)}{n!}.$$

$$f(z) = f(z_0) + \frac{f'(z_0)}{1!}(z-z_0) + \frac{f''(z_0)}{2!}(z-z_0)^2 + \dots + \frac{f^{(n)}(z_0)}{n!}(z-z_0)^n + R(z).$$

$$e^z = \sum_{n=0}^{\infty}\frac{z^n}{n!}.$$

$$\sin z = \sum_{n=0}^{\infty}(-1)^n\frac{z^{2n+1}}{(2n+1)!}, \quad \cos z = \sum_{n=0}^{\infty}(-1)^n\frac{z^{2n}}{(2n)!}$$

$$\frac{1}{1-z} = \sum_{n=0}^{\infty} z^n, \quad \frac{1}{1+z} = \sum_{n=0}^{\infty}(-z)^n.$$

$$(1+z)^\alpha = 1 + \alpha z + \frac{\alpha(\alpha-1)}{2!}z^2 + \dots$$

4、Laurent Series.

$$f(z) = \sum_{n=-\infty}^{+\infty} C_n (z-z_0)^n, \quad C_n = \frac{1}{2\pi i}\int_c \frac{f(z)}{(z-z_0)^{n+1}} dz.$$

求解 Laurent Series, 一般使用间接法进行求解.

当 $n=-1$时, $C_{-1} = \frac{1}{2\pi i}\int_c \frac{f(z)}{1} dz$, 则 $\int f(z) = 2\pi i\, \underbrace{C_{-1}}_{Res}$.

五、Residues and Its Application.

1、singular point → isolated.

Three type of isolated singular points 对函数进行洛朗级数展开
- 如果在这一点有负幂次项，pole of order k, k=1 ⇒ simple pole ~~essential singular point~~
- 无穷多个负幂次项，essential singular point.
- 无负幂次项，removed singular point.

判断 pole of order m 的方法. $f(z)=\dfrac{\varphi(z)}{(z-z_0)^m}$, $\varphi(z_0)\neq 0$ 且解析，那么 $f(z)$ 存在 z_0 处 m 阶 pole.

↓ 但是，函数的分母并不总是差值的形式.

引入零点，$f^{(m)}(z_0)\neq 0$ 但是 $f(z_0)=\cdots=f^{(m-1)}(z_0)=0$, z_0 是 zero of order m.

↓

判断零点，$f(z)=(z-z_0)^m\varphi(z)$, $\varphi(z_0)\neq 0$, $f(z)$ 在 z_0 时，zero of order m.

$\dfrac{f(z)}{\varphi(z)}$, $f(z_0)\neq 0$, $\varphi(z)$ 在 z_0 处 zero of order m, $\dfrac{f(z)}{\varphi(z)}$ 是 pole of order m.

(上述 $f(z_0)\neq 0$, 如果 $f(z_0)=0$, 那么 $f(z)$ 在 z_0 处也有零点，将零点以 $(z-z_0)^n\phi(z)=f(z)$ 的形式全部提取出，再与分母抵消，得到具体形式.)

2、Residues.

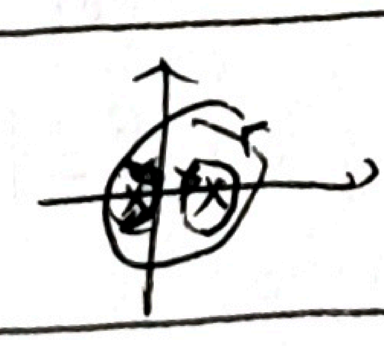
$$\int_C f(z)\,dz = 2\pi i\,C_{-1} = 2\pi i \operatorname*{Res}_{z=z_0} f(z),$$ [C_{-1}, 洛朗级数负1次幂前的系数].

求留数 找 C_{-1}

① 求洛朗级数，-1 次幂前面的系数即为 Res.

② Cauchy's ~~resti~~ residue theorem.

$$\int_C f(z)\,dz = 2\pi i \sum_{i=1}^{n} \operatorname*{Res}_{z=z_i} f(z).$$



③ 奇点太多太多，一个一个求十分困难，可转换一种形式奇点.

$$\int_C f(z)\,dz = 2\pi i \operatorname*{Res}_{z=0}[\varphi(z)],\quad \varphi(z)=\frac{1}{z^2}f\left(\frac{1}{z}\right)$$

(如果反转过来 0 不是 $\varphi(z)$ 的奇点，$\operatorname*{Res}_{z=0}\varphi(z)=0$).

避开求级数.

④ 洛朗级数并不好展开 ($\dfrac{1}{(z-z_0)^n}$ 的洛朗级数).

如果 $f(z)$ 在 z_0 处有 m 阶 pole, 则 $f(z)=\dfrac{\varphi(z)}{(z-z_0)^m}$

那么. $\operatorname*{Res}_{z=z_0} f(z) = \dfrac{\varphi^{(m-1)}(z_0)}{(m-1)!}$ ← 使用高阶导数公式推导.

⑤ 上述过程的特殊情况，一阶极点，求解.

$f(z)=\dfrac{\varphi(z)}{h(z)}$, $\varphi(z_0)\neq 0$, $h(z)$ 在 z_0 处 1 阶零点，或者说，$f(z)$ 在 z_0 是 Simple pole.

那么有 $\operatorname*{Res}_{z=z_0} = \dfrac{\varphi(z_0)}{h'(z_0)}$

工程数学 — 数学物理方法.

一、Equation of mathematical Physics and Problems for Defining Solutions.

二、Classification and Simplification for Linear Second order PDEs.

三、Integral method on Characteristics.

四、The method of Separation of Variable on Finite Region.

五、Special Function.

六、Integral Transformations.

一、数学物理方程.

PDEs条件
- 初值
  - $u|_{t=t_0} = f(\varphi)$.
  - $u_t \ldots |_{t=t_0} = f(\varphi)$.
- 边值
  - ① $u(x,t)|_{x=0} = u(x,t)|_{x=l} = 0$.
  - ② $\frac{\partial u}{\partial n}|_{x,y,z=?} = u_0$.
  - ③ 混合 ①, ②.

三类PDE方程
- ① wave equation. $\frac{\partial^2 u}{\partial t^2} = a^2\nabla^2 u + F(x,t)$.
- ② heat equation. $\frac{\partial u}{\partial t} = a^2\nabla^2 u + F(x,t)$.
- ③ Laplace equation. $\nabla^2 u = F(x)$.

二、PDEs的化简.

$$Au_{xx} + Bu_{xy} + Cu_{yy} + Du_x + Eu_y + Fu = G.$$

$\to A\left(\frac{dy}{dx}\right)^2 - B\frac{dy}{dx} + C = 0$.

$\to \frac{dy}{dx} = \frac{B \pm \sqrt{B^2-4AC}}{2A}$.

$\Rightarrow$
- $\Delta = 0$, $\frac{dy}{dx} = \frac{B}{2A}$, 求得一个$\frac{dy}{dx}$, 解出 $\underbrace{f(x,y)}_{\xi} = const.$, 令 $\eta = y$ 即可.
- $\Delta > 0$, 得到两个方程(ODEs), 解出 $f(x,y) = const$, $g(x,y) = const$, $\begin{cases} \xi = f(x,y) \\ \eta = g(x,y) \end{cases}$.
- $\Delta < 0$, 得到共轭复根, $\phi(x,y) \pm i\psi(x,y) = const$, 令 $\begin{cases} \xi = \frac{\Phi + \Psi}{2} = \phi(x,y) \\ \eta = \frac{\Phi - \Psi}{2i} = \psi(x,y) \end{cases}$,
  即令实部和虚部分别为 $\underbrace{\phi(x,y)}_{\xi}$ 和 $\underbrace{\psi(x,y)}_{\eta}$

随后利用偏导性质, 将变量全换为 ξ 和 η, 即可化简.

三、 D'Alembert formula.

a) 无界弦振动.

$$\begin{cases} u_{tt}-a^2u_{xx}=0, & -\infty<x<+\infty,\ t>0. \\ u(x,0)=\phi(x), & -\infty<x<+\infty \\ u_t(x,0)=\psi(x), & -\infty<x<+\infty. \end{cases}$$

$$\Rightarrow \begin{cases} \xi=x-at \\ y=x+at. \end{cases} \Rightarrow u_{\xi y}=0. \Rightarrow \underbrace{u(\xi,y)=f(\xi)+g(y).}_{\Downarrow}$$

$$\cancel{u(x,y)}\ \underbrace{u(x,t)=f(x-at)+g(x+at).}$$

然后将 $u(x,t)=f(x-at)+g(x+at)$ 用初值条件判断

$$\boxed{u(x,t)=\frac{1}{2}\left(\phi(x-at)+\phi(x+at)\right)+\frac{1}{2a}\int_{x-at}^{x+at}\psi(z)\,dz.}$$

b) 有界(半无界弦振动). + 第一类边值条件.

i)

$$\begin{cases} u_{tt}-a^2u_{xx}=0, & 0<x<+\infty,\ t>0 \\ u(x,0)=\phi(x), & 0\le x<+\infty \\ u_t(x,0)=\psi(x), & 0\le x<+\infty. \\ u(0,t)=0, & t\ge 0. \end{cases}$$

半无界弦 + 边值条件(第一类).

如果 D'Alembert formula 依然适用.

$$\begin{cases} u(0,t)=\frac{1}{2}\left(\underbrace{\phi(-at)+\phi(at)}\right)+\frac{1}{2a}\int_{-at}^{at}\underbrace{\psi(z)}dz=0. \\ \Rightarrow \phi(z)\text{ 和 }\psi(z)\text{ 均为奇函数}. \end{cases}$$

则只需要对 $\psi(x)$ 和 $\phi(x)$ 进行延拓(奇延拓), 然后使用 D'Alembert formula.

并且在后需求解中, 对 x 进行讨论.

ii) 半无界弦振动 + 第二类边值条件.

$$\begin{cases} u_{tt}-a^2u_{xx}=0, & 0<x<+\infty,\ t>0 \\ u(x,0)=\phi(x), & 0\le x<+\infty \\ u_t(x,0)=\psi(x), & 0\le x<+\infty \\ u_x(0,t)=0, & t\ge 0. \end{cases}$$

仍使用 D'Alembert. formula.

$$\begin{cases} \frac{1}{2}\left(\phi'(at)+\phi'(-at)\right)+\frac{1}{2a}\left(\psi(at)-\psi(-at)\right). \\ \text{那么 }\phi(x)\text{ 和 }\psi(x)\text{ 应都为偶函数, 才能满足条件}. \end{cases}$$

对原条件进行偶延拓, 随后进行讨论求解.

四、分离变量法

a) 一维齐次方程

i) 有界区域一维弦振动

$$\begin{cases} \frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2}, & 0<x<l,\ t>0 \\ u(0,t)=u(l,t)=0, & t\geq 0 \\ u(x,0)=f(x),\ \frac{\partial u}{\partial t}\Big|_{t=0}=\varphi(x), & 0\leq x\leq l \end{cases}$$

令 $u(x,t)=X(x)T(t)$. $\Rightarrow \left\{ \frac{T''}{a^2T} = \frac{X''}{X} = -\lambda \right.$ $\boxed{\lambda>0}$

随后可得到 $\begin{cases} T''+\lambda a^2 T=0 \\ X''+\lambda X=0 \end{cases}$ 从边界条件入手，对λ进行特征根讨论.

得到λ的特征值和对应的 $X(x)$，再代入 $T(t)$ 进行求解.

随后线性组合 ~~X(x) 和 T(t)~~ $X(x)T(t)$ 构成方程的解集.

随后代入初值条件进行求解，变成 Fourier 级数的展开问题.（两次 Fourier 展开）

ex. $\sum_{n=1}^{\infty} \underbrace{b_n \frac{a n\pi}{l}}_{\text{Fourier系数}} \sin\frac{n\pi}{l}x = \varphi(x)$，$b_n\frac{an\pi}{l} = \frac{2}{l}\int_0^l \varphi(x)\sin\frac{n\pi}{l}x\,dx$

半基波周期.

ii) 有界区域一维热传导

$$\begin{cases} \frac{\partial u}{\partial t} = a^2\frac{\partial^2 u}{\partial x^2}, & 0<x<l,\ t>0 \\ \frac{\partial u}{\partial x}(0,t)=\frac{\partial u}{\partial x}(l,t)=0, & t\geq 0 \\ u(x,0)=f(x), & 0\leq x\leq l \end{cases}$$

$\boxed{\lambda\geq 0}$

先分离变量，令 $\frac{X''(x)}{X(x)} = \frac{T'(t)}{a^2T(t)} = -\lambda \Rightarrow \begin{cases} X''(x)+\lambda X(x)=0 \\ T'(t)+\lambda a^2 T(t)=0 \end{cases}$

同样，从边值条件入手，讨论λ取值，获得 $X(x)$ 和λ的序列值.

同样去除线性组合，有

$\sum_{n=0}^{\infty} C_n\cos\frac{n\pi}{l}x = f(x)$. $\begin{cases} C_0 = \frac{1}{2}\cdot\frac{2}{l}\int_0^l f(x)dx \\ C_n = \frac{2}{l}\int_0^l f(x)\cos\frac{n\pi x}{l}dx \end{cases}$ 有时直接用 C_n 不好处.

解出 C_n，$u(x,t) = C_0 + \sum_{n=1}^{\infty} C_n e^{-(\frac{n\pi a}{l})^2 t}\cos\frac{n\pi x}{l}$，~~[illegible]~~（一次 Fourier 展开）.

b) 二维方程.

i) 矩形域 Laplace 方程.

$$\begin{cases} \Delta u = u_{xx} + u_{yy} = 0, & 0<x<a,\ 0<y<b. \\ u_x(0,y)=0,\ u_x(a,y)=0, & 0\le y\le b \\ u(x,0)=f(x),\ u(x,b)=0, & 0\le x\le a. \end{cases}$$

$\boxed{\lambda \ge 0}$

分离变量，令 $\frac{X''}{X} = -\frac{Y''}{Y} = -\lambda$，随后对比选看起来比较简单的边值条件进行代入.

中途在讨论 Y 时，可能出现 $Y'' - \left(\frac{n\pi}{a}\right)^2 Y = 0$，此时注意对 $n=0$ 的讨论.

总和的 $u(x,t) = a_0 y + b_0 + \sum_{n=1}^{\infty}\left(a_n e^{\frac{n\pi}{a}y} + b_n e^{-\frac{n\pi}{a}y}\right)\cos\frac{n\pi}{a}x.$

ii) 圆形区域 Laplace 方程.

$$u_{xx}+u_{yy}=0 \xrightarrow[y=\rho\sin\theta]{x=\rho\cos\theta} \begin{cases} \frac{\partial^2 u}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial u}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2 u}{\partial \theta^2} = 0, & \rho<R,\ 0\le\theta\le 2\pi. \\ u(\rho,\theta) = u(\rho,\theta+2\pi), & \rho\le R \\ u(R,\theta) = a\cos\theta, & 0\le\theta\le 2\pi. \\ u(0,\theta) < +\infty, & 0\le\theta\le 2\pi. \end{cases}$$

分离变量，令

$$\frac{\rho^2 P''(\rho)}{P(\rho)} + \frac{\rho P'(\rho)}{P(\rho)} = -\frac{\Theta''(\theta)}{\Theta(\theta)} = \lambda.$$

$\boxed{\lambda \ge 0}$，$n=0,1,2\cdots$

ODE.

$$\underbrace{\rho^2 P''(\rho)}_{\text{逐阶降次}} + \rho P'(\rho) - n^2 P(\rho) = 0$$

这类ODE为欧拉ODE，设 $P(\rho)=\rho^m$ 代入求 m，然后再线性组合.

上述需要讨论 $n=0$ 时，$\rho^2 P''(\rho) + \rho P'(\rho) = 0 \Rightarrow [\rho P'(\rho)]' = 0 \Rightarrow P_0(\rho) = C_0\ln\rho + D_0.$

$n\ge 1$ 时，$P_n(\rho) = C_n\rho^{-n} + D_n\rho^n$

当 $\rho=0$，$u(0,\theta)<+\infty$，则 $C_0=0$，$C_n=0$，$P(\rho) = D_0 + \sum_{n=1}^{\infty} D_n\rho^n = \sum_{n=0}^{\infty} C_n\rho^n.$

c) Sturm–Liouville ~~eigen~~ eigenvalue problem.

$$f(x) = \sum_{n=1}^{\infty} f_n u_n(x).$$

$$f_n = \frac{\int_a^b s(x) f(x) u_n(x)\,dx}{\int_a^b s(x) u_n^2(x)\,dx} \quad (n=1,2,\cdots).$$

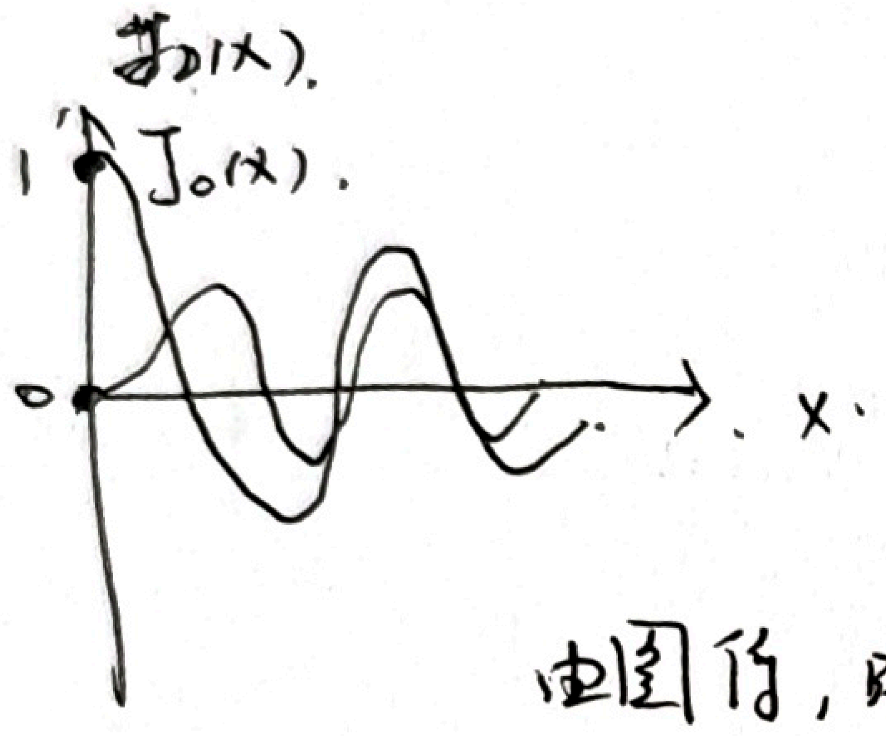
五、特殊函数.

A. Bessel Function.

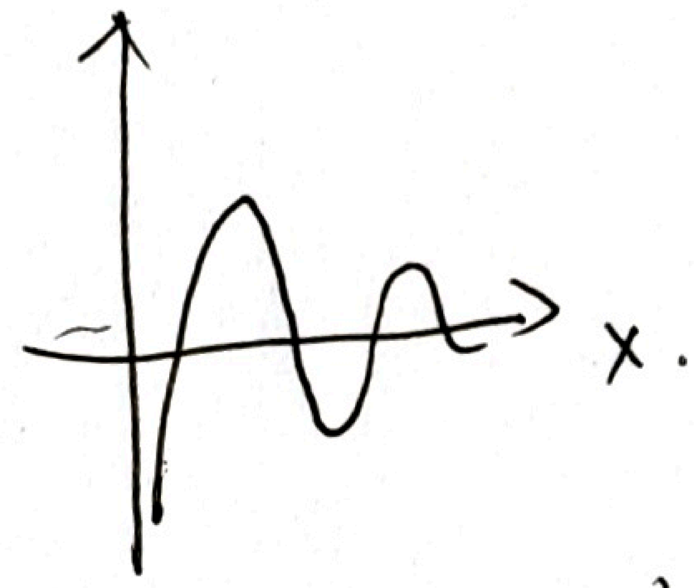
$$x^2y''(x)+xy'(x)+(x^2-\nu^2)y(x)=0$$, ν阶 Bessel equation.

Solution: $y(x)=C_1J_\nu(x)+C_2Y_\nu(x)$.

$J_\nu(x)$



$Y_\nu(x)$



由图得，只有 $J_0(x)\big|_{x=0}=1$，其它 $J_\nu(x)\big|_{x=0}=0$.

$J_\nu(x)$的性质
$$\begin{cases}\dfrac{d}{dx}[x^\nu J_\nu(x)]=x^\nu J_{\nu-1}(x)\\[2ex]\dfrac{d}{dx}[x^{-\nu}J_\nu(x)]=-x^{-\nu}J_{\nu+1}(x).\end{cases}$$

B. Legendre polynomial

$$(1-x^2)\frac{d^2P}{dx^2}-2x\frac{dP}{dx}+\nu(\nu+1)P(x)=0.$$

如果 ν 不是整数. 得 $P(x)=a_0p_\nu(x)+a_1q_\nu(x)$.

如果 ν 是整数. $P_n(x)=C_1\underline{P_n(x)}+C_2\underline{Q_n(x)}$.

第一类勒让德多项式. 第二类勒让德函数.

$P_n(x)$的性质
$$\begin{cases}P_0(x)=1\\P_1(x)=x.\\(n+1)P_{n+1}(x)-(2n+1)xP_n(x)+nP_{n-1}(x)=0.\\\displaystyle\int_{-1}^{1}P_n(x)P_m(x)=\begin{cases}0, & n\neq m.\\\dfrac{2}{2n+1}, & n=m.\end{cases}\end{cases}$$

六. 积分变换.

A. Fourier integral transformation.

$$F(\lambda)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}f(x)e^{i\lambda x}dx.$$

$$f(x)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}F(\lambda)e^{-i\lambda x}d\lambda.$$

B. Laplace integral transformation.

$$\boxed{\begin{aligned} L(s) &= \int_{0}^{+\infty}f(t)e^{-st}dt. \\ f(t) &= \frac{1}{2\pi i}\int_{-\infty}^{+\infty}L(s)e^{st}ds. \end{aligned}}$$

对函数时间进行 Laplace 变换.

$$\boxed{\begin{aligned} f'(t) &\xleftrightarrow{L} sL(s)-f(0^+). \\ f''(t) &\xleftrightarrow{L} s^2L(s)-sf(0^+)-f'(0^+). \end{aligned}}$$

$$t^n \xleftrightarrow{L} \frac{n!}{s^{n+1}}$$

$$\cos kt \xleftrightarrow{L} \frac{s}{s^2+k^2}.$$

$$\sin kt \xleftrightarrow{L} \frac{k}{s^2+k^2}.$$

$$\cancel{\int_0^{+\infty}\frac{\sin t}{t}} \quad \int_0^{+\infty}\frac{\sin t}{t}dt=\frac{\pi}{2}$$