大学物理(二)

一、热学

1、气体动理论.

2、热力学第一定律.

3、热力学第二定律.

1、气体动理论.

— <理想气体状态方程>.

$PV = nRT = NkT$

STP: $T = 273.15\,k$, $P = 1.00\,atm$.

absolute pressure 绝对压强

gauge pressure 计量压强 (压力大于标准大气压[1 atm]的压强).

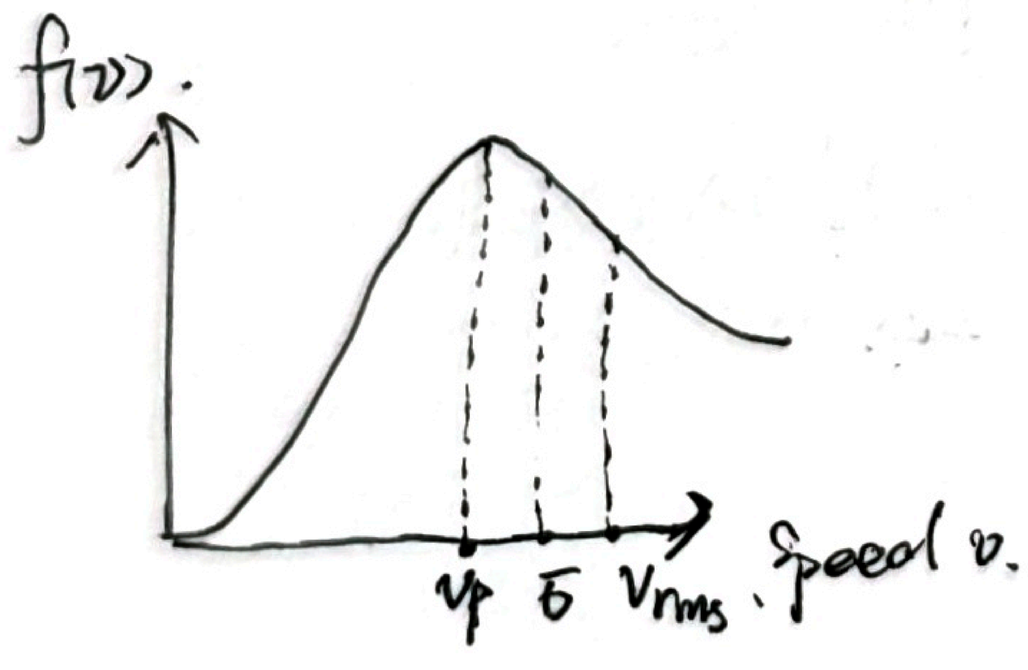
gauge pressure of 8.7 atm. $P = (8.7+1)\,atm = 9.7\,atm$.

$\underline{mole}_{n} \longleftrightarrow \underline{molecules}_{N}$, molecules = N_A × mole.

— < average translational kinetic energy >.

$\bar{K} = \frac{1}{2} m \bar{v}^2 = \frac{3}{2} kT$. 单个分子的平均动能.

— < Maxwell Distribution >.



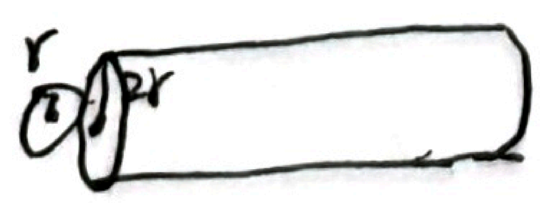
$f(v)\,dv = dN$.

$v_p = \sqrt{2\frac{kT}{m}}$.

$\bar{v} = \sqrt{\frac{8}{\pi}\frac{kT}{m}} \approx 1.60\sqrt{\frac{kT}{m}}$.

$v_{rms} = \sqrt{3\frac{kT}{m}}$ (可用$\bar{K}$求得).

— < Mean Free Path >.

想象  的推导过程.

$l_m = \frac{1}{\pi(2r)^2\sqrt{2}\left(\frac{N}{V}\right)}$.

neon (Ne)
argon (Ar) 氩气.
nitrogen (N_2)
helium (He).
oxygen (O_2).
hydrogen (H_2).
~~co~~ carbon dioxide (CO_2) carbon monoxide (C

2、热力学第一定律.

< Internal Energy >.

是气体分子的动能、势能总和，但由于理想气体不考虑势能因素(分子间或分子内作用力)，
所以再加上对自由度的考虑(可由平均分子动能导出)

$U = \frac{i}{2} nRT = \frac{i}{2} NkT$ (monatomic gas $i=3$, diatomic gas $i=5$, triatomic gas $i=6$.)

<Specific Heat>.

$Q = mc\Delta T$. 小写 c，$c = \frac{Q}{m\Delta T}$，m 为物体总质量，此公式一般适用于固体。

$Q = nC\Delta T$ (Molar Specific Heats for Gases).

<The First Law of Thermodynamics>.

$\Delta U = Q - W$.

a) 利用热力学第一定律表达摩尔比热容相关关系 (两种热过程).

process 1. Constant V.

$Q = nC_V\Delta T$, ~~W=~~ $\Delta U = Q - W = Q \Rightarrow C_V = \frac{i}{2}R$

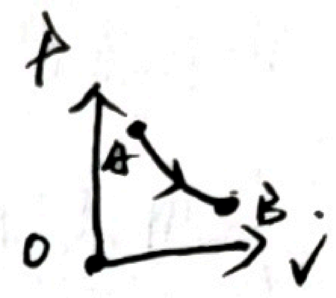
process 2. Constant P.

$Q = nC_p\Delta T$, $\Delta U = Q - W \Rightarrow C_p = C_V + R$.

$\Rightarrow \gamma = \frac{C_p}{C_V} = \frac{2+i}{i}$.

b) 其余两种热过程.

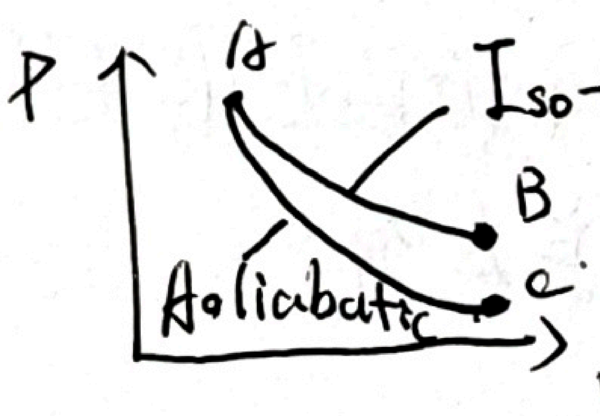
Process 3. Isothermal ($\Delta T = 0$).



$C_T = \infty$

$\Delta U = Q - W = 0$, $Q = W = \int_{V_A}^{V_B} P\,dV \xrightarrow{PV=nRT} \int_{V_A}^{V_B} \frac{nRT}{V}\,dV = nRT\ln\frac{V_B}{V_A}$.

Process 4. Adiabatic ($Q = 0$)



$\Delta U = Q - W = -W$.

$$\left\{\begin{array}{l} dU = -P\,dV \\ dU = nC_V\,dT \\ PV = nRT \end{array}\right. \Rightarrow PV^\gamma = \text{Constant}, \text{ where } \gamma = \frac{2+i}{i}$$

求做功的方式：(结合上述图示示意，仅描述 A→B 过程 adiabatic)

i) 积分. $W = \int_{V_A}^{V_B} P\,dV = \int_{V_A}^{V_B} \frac{C}{V^\gamma}\,dV$.

$= C\int_{V_A}^{V_B} V^{-\gamma}\,dV = C\frac{1}{-\gamma+1}\left(V_B^{-\gamma+1} - V_A^{-\gamma+1}\right)$.

ii) 第一定律 $W = -\Delta U = -nC_V\Delta T$, $T_1 \to T_2$ 时段.

$\xrightarrow{\gamma = \frac{C_p}{C_V}} W = -\frac{P_2V_2 - P_1V_1}{\gamma - 1}$

c) Free expansion.

Adiabatic 的一种，对气体既没有吸放热，也没有做功.

这个过程迅速，不平衡的，不是 quasistatic 的.

$Q = W = \Delta U = 0$.

3. Second Law of Thermodynamics.

<heat engine>.

$$|Q_H| = |W| + |Q_L|.$$

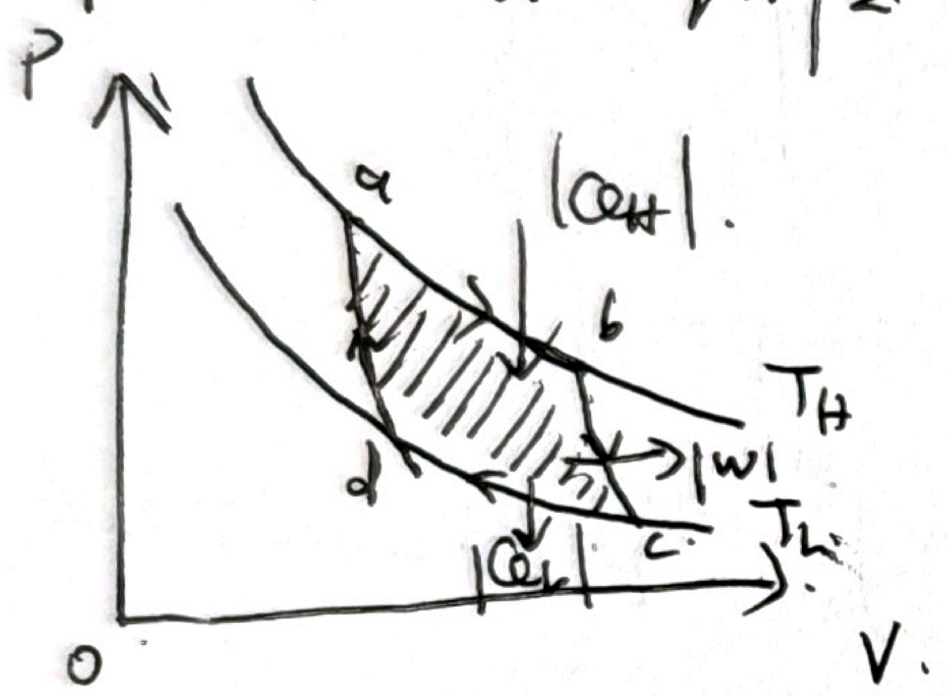
$|Q_H|$：投入 heat engine 的所有热量，包括所有的$\boxed{\text{吸热过程}}$。
切记包括所有的吸热过程，Carnot's Engine 是特例，所以才能一致。
因为其本质只有一段吸热过程。

$|W|$：热机做的所有功之和的绝对值，也切记包括所有功。

$$e = \frac{|W|}{|Q_H|} = \frac{|Q_H| - |Q_L|}{|Q_H|} = 1 - \frac{|Q_L|}{|Q_H|}.$$

<Reversible Process and Carnot's Engine>.

可逆过程在真实生活中是不存在的，只有 Irreversible Process.



两段等温，两段绝热过程。

使用其之间的对应关系可以求得 e.

$$e = 1 - \frac{|Q_L|}{|Q_H|} = 1 - \frac{T_L}{T_H}.$$

卡诺定理：相同高温热源和低温热源工作的一切可逆热机，无论使用何种工作物质，总会有

$$e = 1 - \frac{T_L}{T_H}.$$

而它也是同等不可逆热机的最大效率。 $e = 1 - \frac{T_L}{T_H}$.

判断热机可逆性需要用 entropy. 在 P-V 图中不可直接套用 $\cancel{e = 1 - \frac{T_L}{T_H}}$，除非是卡诺循环，其它的过程还是先老实算 Q_H 和 W. 注意每个 stage.

* < reverse of a heat engine >.

Coefficient of performance (COP).

$$\text{空调}.\ COP = \frac{Q_L}{W} = \frac{Q_L}{Q_H - Q_L} \sim \frac{T_L}{T_H - T_L} \ \text{理想}$$

$$\text{heat pump}\quad COP = \frac{Q_H}{Q_H - Q_L} \sim \frac{T_H}{T_H - T_L}.$$

二、光学.

A. Interference of Light.

B. Diffranction of light.

C. Polarization of Light.

A. Interference of Light.

Critical angle 临界角，发生全反射时最小入射角.

angle of incidence 入射角

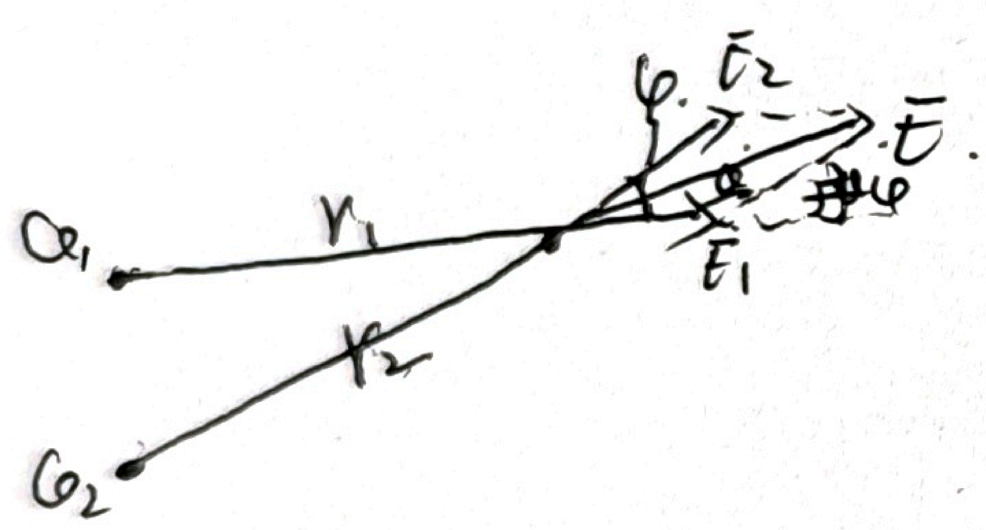
入射.

< light speed>.

$$c=\frac{1}{\sqrt{\varepsilon_0\mu_0}}=3\times10^8\ m/s.$$

< Huygen's principle>.

< Two - ~~sou~~source interference>.



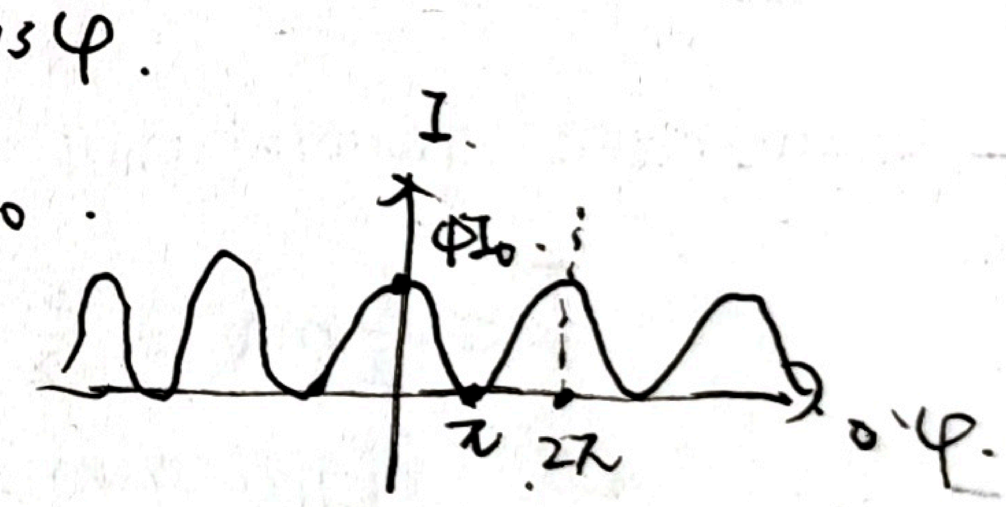
$$\vec{E}^2=E_1^2+E_2^2-2E_1E_2\cos\theta$$

$$\xRightarrow{\theta=\pi-\varphi} E_1^2+E_2^2+2E_1E_2\cos\varphi.$$

$$\Rightarrow I=I_1+I_2+2\sqrt{I_1I_2}\cos\varphi.$$

如果条件再强一些 O_1, O_2 两列波，波的强度一致，为 I_0.

$$I=2I_0+2I_0\cos\varphi=2I_0(1+\cos\varphi)=4I_0\cos^2\frac{\varphi}{2}.$$



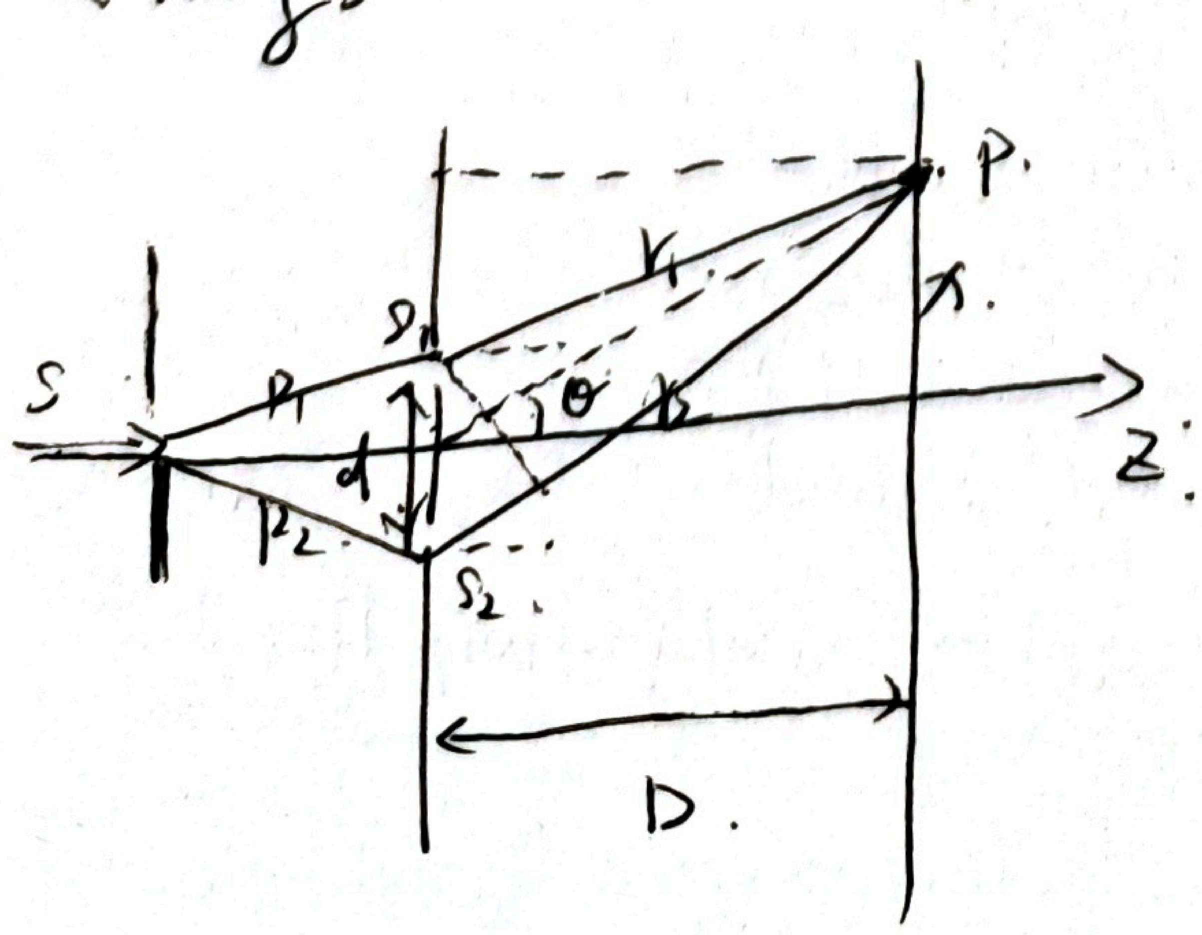
若光程差来相位差，$\Delta\varphi=\frac{2\pi}{\lambda}(r_1-r_2)$. $\Rightarrow$
$$\begin{cases} r_1-r_2=k\lambda, & \text{极大} \\ r_1-r_2=(k+\frac{1}{2})\lambda, & \text{极小.} \\ k=0,\pm1,\pm2\ldots\ldots \end{cases}$$

< Optical path length>.

$$L=\sum_{i=1}^{m}n_il_i.$$

$$\Delta\varphi=\frac{2\pi}{\lambda}\Delta L. \Rightarrow \Delta\varphi=\frac{2\pi}{\lambda}\delta.$$

< Young's two-slit interference experiment>.



$$r_2-r_1\approx d\sin\theta\approx d\tan\theta=\frac{dx}{D}.$$

$$\Delta\varphi=\frac{2\pi d}{\lambda}\sin\theta=\frac{2\pi d}{\lambda D}x.$$

$$\begin{cases} \frac{2\pi d}{\lambda}\sin\theta=\frac{2\pi d}{\lambda}\frac{x}{D}=2k\pi,\ x=k\frac{D}{d}\lambda,\ \Delta x=\frac{D}{d}\lambda;\ \text{亮}. \\ \frac{2\pi d}{\lambda D}x=(2k+1)\pi,\ x=(k+\frac{1}{2})\frac{D}{d}\lambda,\ \text{暗}. \end{cases}$$

$\begin{cases} \delta = d\sin\theta = k\lambda, & \text{bright fringe} \leftrightarrow \text{constructive interference.} \\ \delta = d\sin\theta = (k+\frac{1}{2})\lambda, & \text{dark fringe} \leftrightarrow \text{destructive interference.} \end{cases}$

$$I = 4I^2\cos^2\frac{\Delta\varphi}{2} \xRightarrow{\Delta\varphi = \frac{2\pi}{\lambda}d\sin\theta} 4I^2\cos^2\left(\frac{\pi}{\lambda}d\sin\theta\right).$$

<Lloyd's mirror>.

考虑半波损失距离（光疏→光密） $\begin{cases} \text{相位上要}\pi. \\ \text{光程上要}\frac{\lambda}{2}. \end{cases}$

<Coherence of light>.

Source → single slit → coherent sources → interference.

若不是相干光， $I = I_1 + I_2 + 2\sqrt{I_1I_2}\underbrace{\langle\cos\Delta\varphi\rangle}_{} = 0$.

n_1 / n_2 / n_3

只有 $n_1<n_2>n_3$ 或 $n_1>n_2<n_3$ 有半波损，$n_1>n_2>n_3$ 和 $n_1<n_2<n_3$ 无半波损.

<Thin-film interference>.

一定尺寸 光程差 等厚(2) 条纹

考虑半波损失

constructive interference. $\delta = 2nt + \boxed{\frac{\lambda}{2}} = m\lambda \ (m=1,2,3\ldots)$.

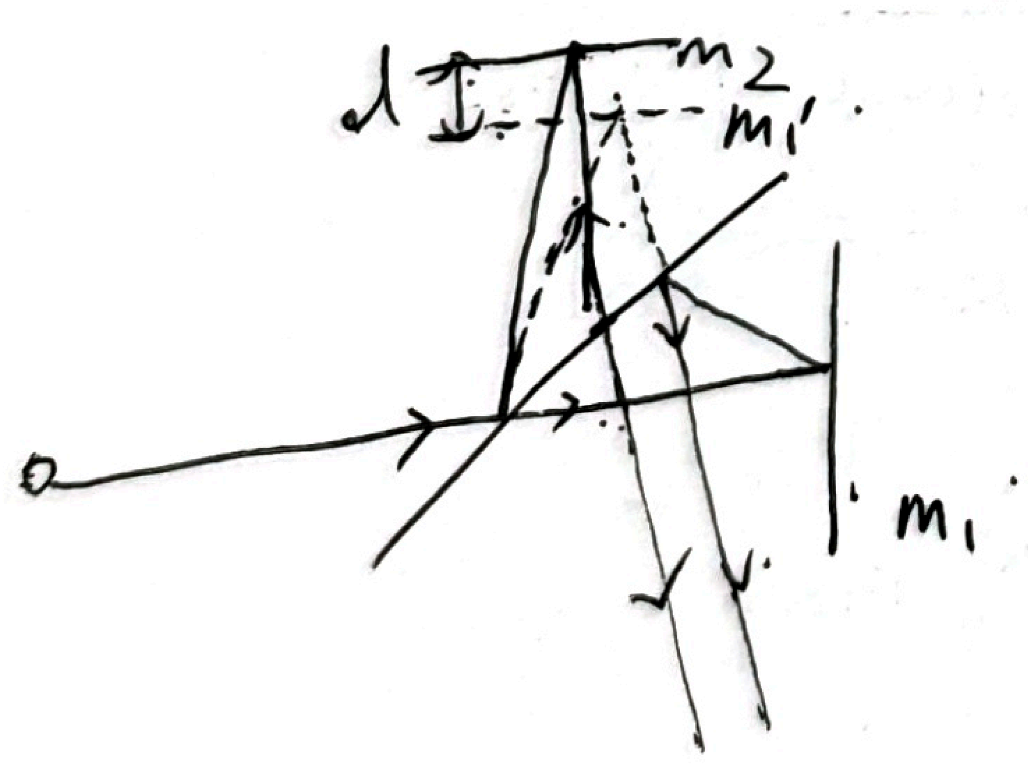
destructive interference $\delta = 2nt + \boxed{\frac{\lambda}{2}} = (2m+1)\frac{\lambda}{2} \ (m=0,1,2\ldots)$. (m=0 时候无意义)

<Newton's ring>.

$$r = \sqrt{R^2-(R-h)^2} = \sqrt{2Rh-h^2} \approx \sqrt{2Rh}.$$

$\begin{cases} 2h+\frac{\lambda}{2} = k\lambda,\ 2h = (k-\frac{1}{2})\lambda,\ r = \sqrt{(k-\frac{1}{2})\lambda R},\ k=1,2,3\ldots, \text{bright.} \\ 2h+\frac{\lambda}{2} = (2k+1)\frac{\lambda}{2},\ 2h = k\lambda,\ r = \sqrt{kR\lambda},\ k=0,1,2\ldots, \text{dark.} \end{cases}$

<Michelson Interferometer>.



$$2d = k\lambda.$$

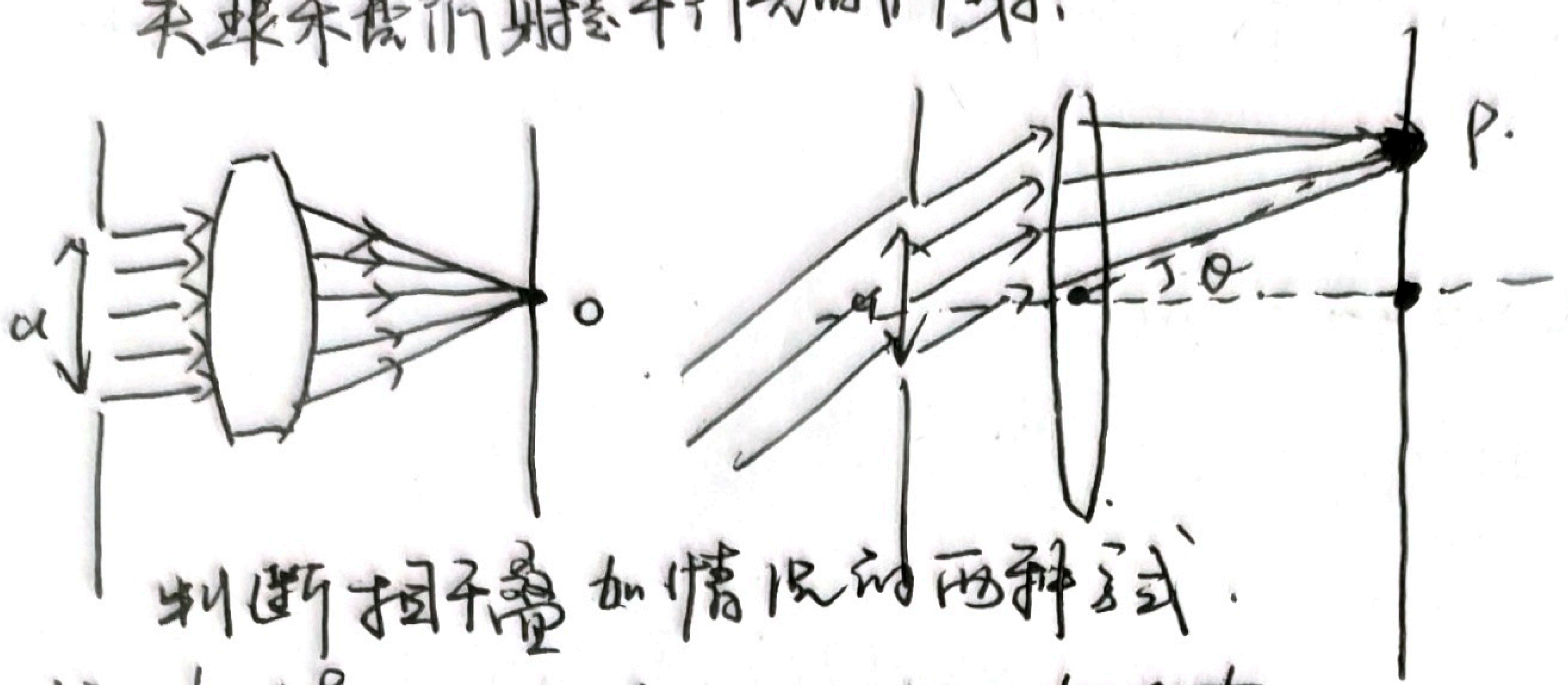
$$d = \frac{k\lambda}{2} \Rightarrow \text{bright; constructive interference.}$$

B. Diffraction of Light.

<Huygens-Fresnel principle>.

<Single-slit Fraunhofer diffraction>.

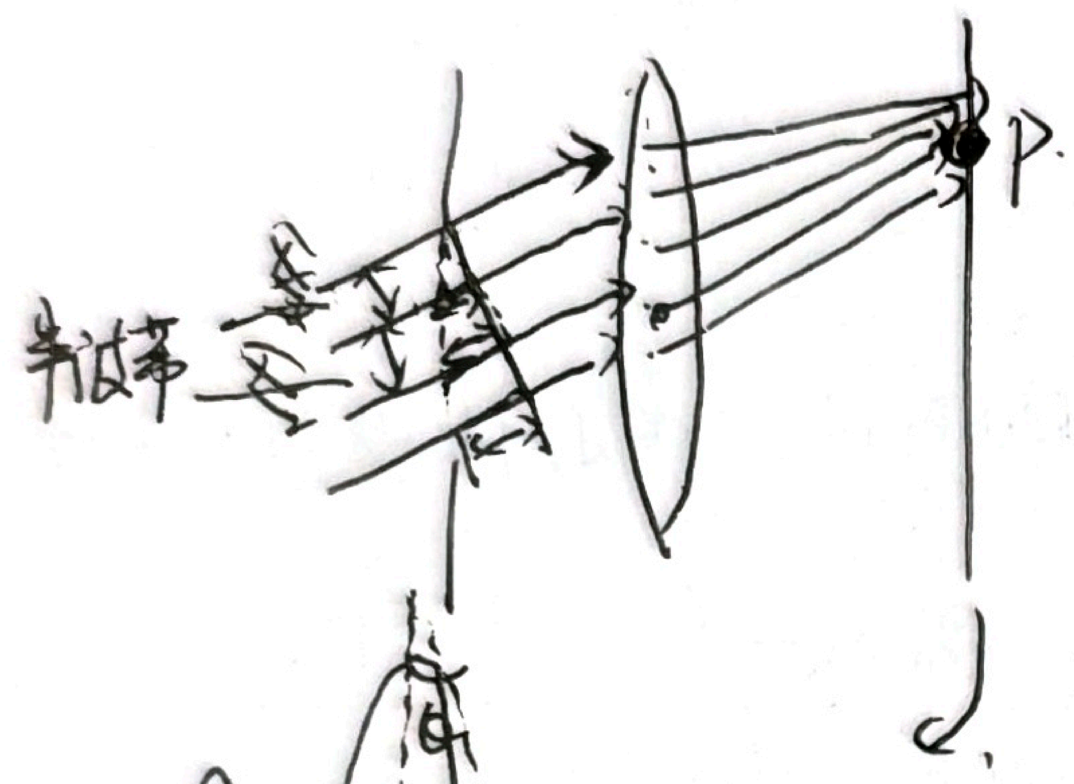
夫琅禾费衍射是平行光的衍射.



透镜具有等光程性.

判断相干叠加情况的两种方式.

i) half-period zone treatment.



$\delta = a\sin\theta$, 每束光和其光程差为 $\frac{\lambda}{2}$ 的光相抵消.

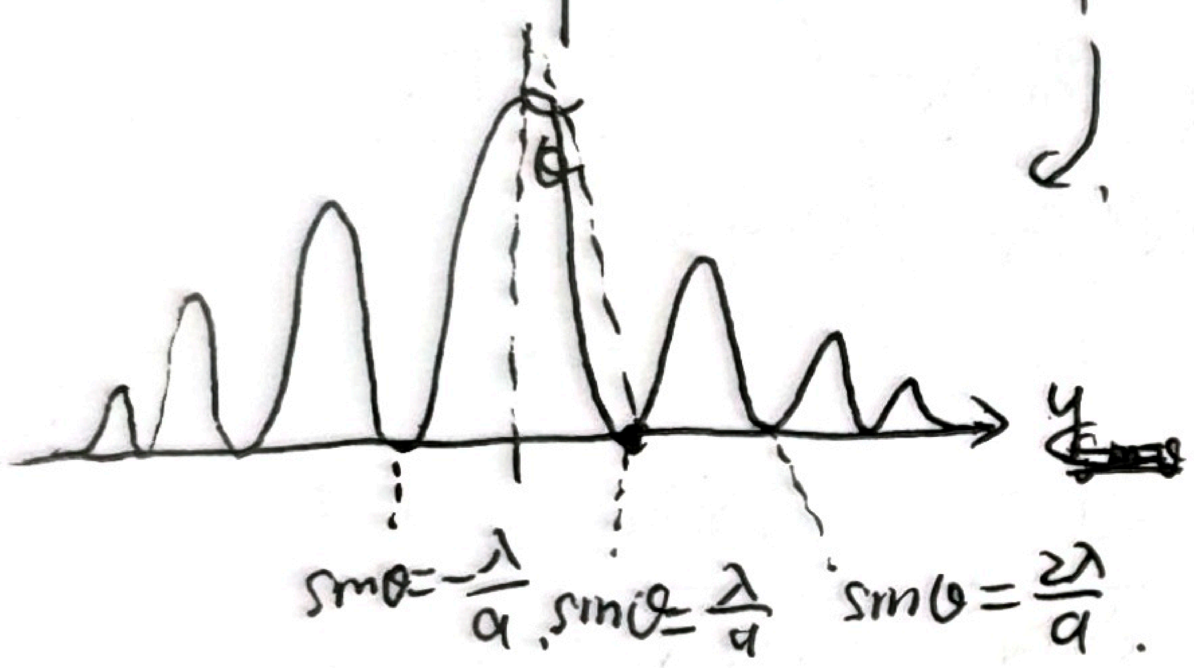
则相邻两半波带的光抵消为暗纹.

若 $\delta = a\sin\theta = N\lambda$.

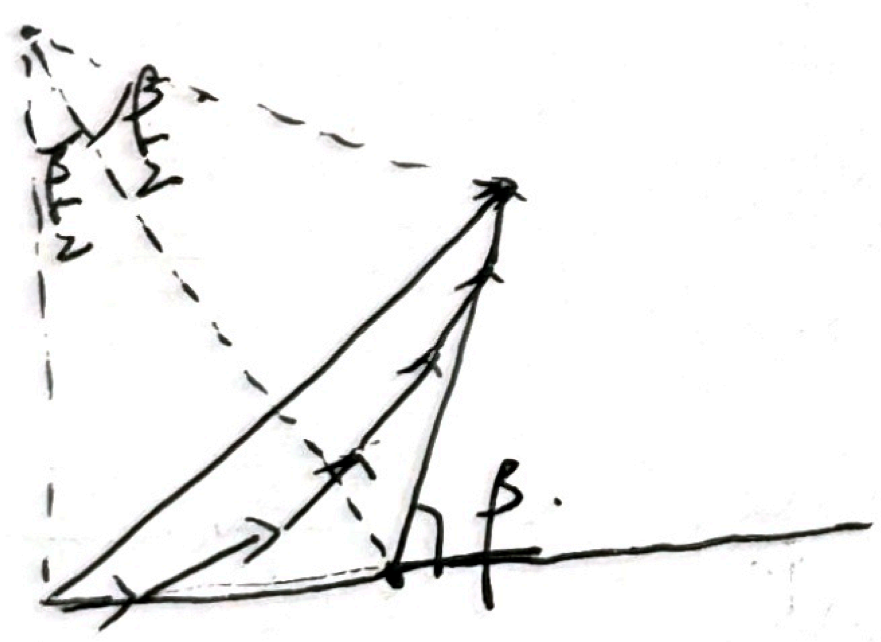
则 $\sin\theta$ 对应的点为暗纹.

若 $\delta = a\sin\theta = (N+\frac{1}{2})\lambda$.

根据此方法, 为亮纹, 但才较多实际不符合.



ii) Phasor treatment.



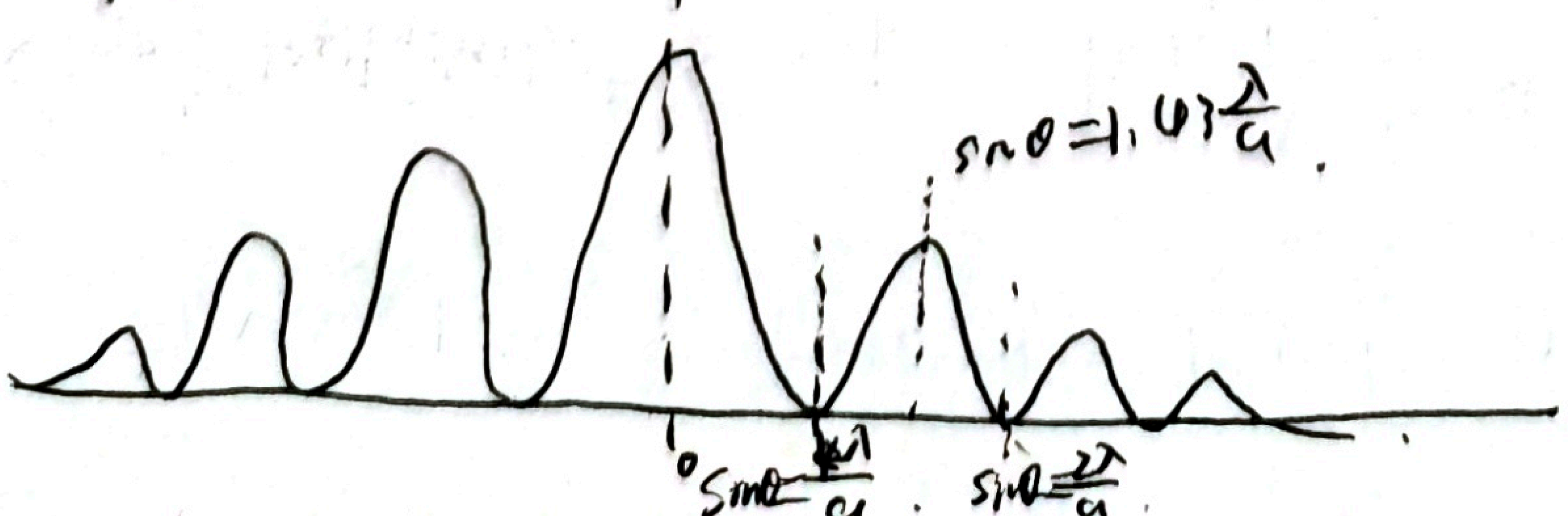
$$\begin{cases} E_0 = 2r\frac{\beta}{2} \\ E = 2r\sin\frac{\beta}{2} \end{cases} \Rightarrow \begin{cases} E = E_0\dfrac{\sin\frac{\beta}{2}}{\frac{\beta}{2}} \\ I = I_0\underbrace{\dfrac{\sin^2\frac{\beta}{2}}{(\frac{\beta}{2})^2}}_{\text{单缝衍射因子}} \end{cases}$$

$$\beta = \frac{2\pi}{\lambda}a\sin\theta.$$

亮: $\frac{d}{d\beta}I = 0$, $\sin\theta = \pm1.43\frac{\lambda}{a}, \pm2.46\frac{\lambda}{a}, \pm3.47\frac{\lambda}{a}\cdots$

$I_1 = 4.7\% I_0$, $I_2 = 1.7\% I_0$, $I_3 = 0.8\% I_0$.

暗: $\begin{cases} \frac{\beta}{2} = 2k\lambda \\ \beta = \frac{2\pi}{\lambda}a\sin\theta \end{cases} \Rightarrow \sin\theta = \frac{k\lambda}{a},\ k=\pm1, \pm2\cdots$

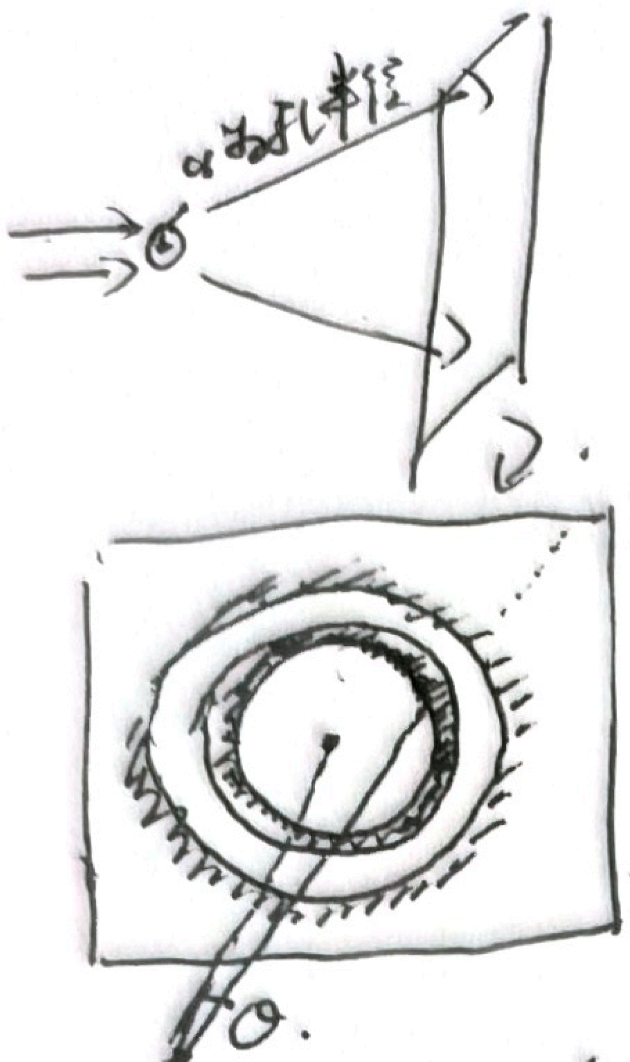


亮斑的角宽度: (相邻暗条纹之间的角距离).

half-width $= \frac{\lambda}{a}$, 可作为衍射强弱的标志.
(零级条纹)

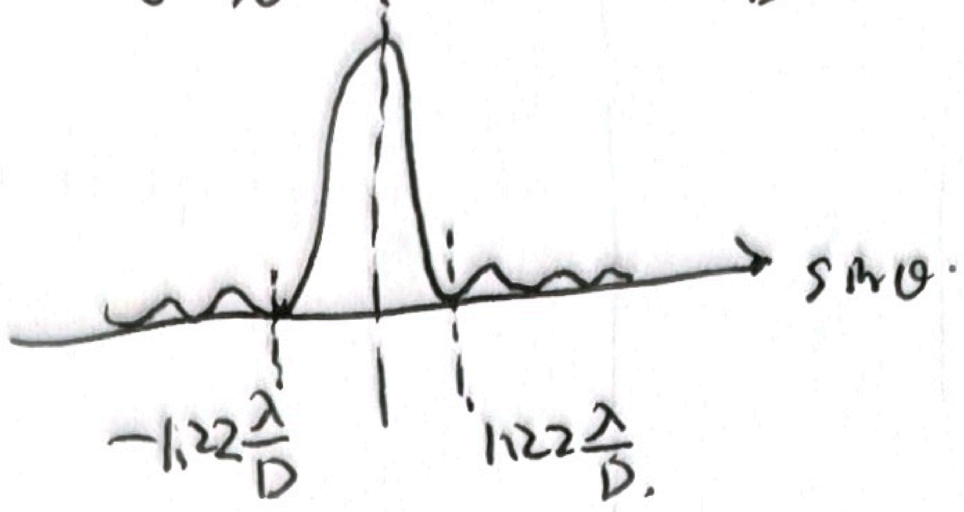
<Circular apertures and resolution of an optical instrument>

夫琅禾费圆孔衍射.



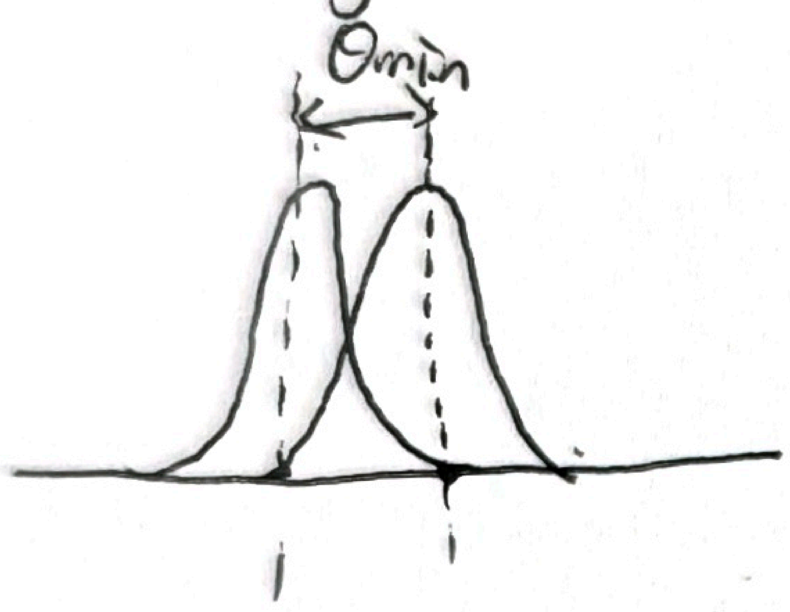
当 $x=\frac{2\pi}{\lambda}a\sin\theta=1.22\pi \Rightarrow (2a)\sin\theta=1.22\lambda$，$\sin\theta=\frac{1.22\lambda}{D}$ 时，$I=0$

$\theta\approx\sin\theta=1.22\frac{\lambda}{D}$，内部亮斑为 Airy 斑.（D为直径）.



由艾里斑，每个像成的不是一个 point，而是一幅衍射图像，如果像点重合，这幅图像就无法分辨.

<Rayleigh criterion> 最小分辨角的标准.



一个圆斑的中心落在另一个圆斑的边缘时，恰好可以分辨.

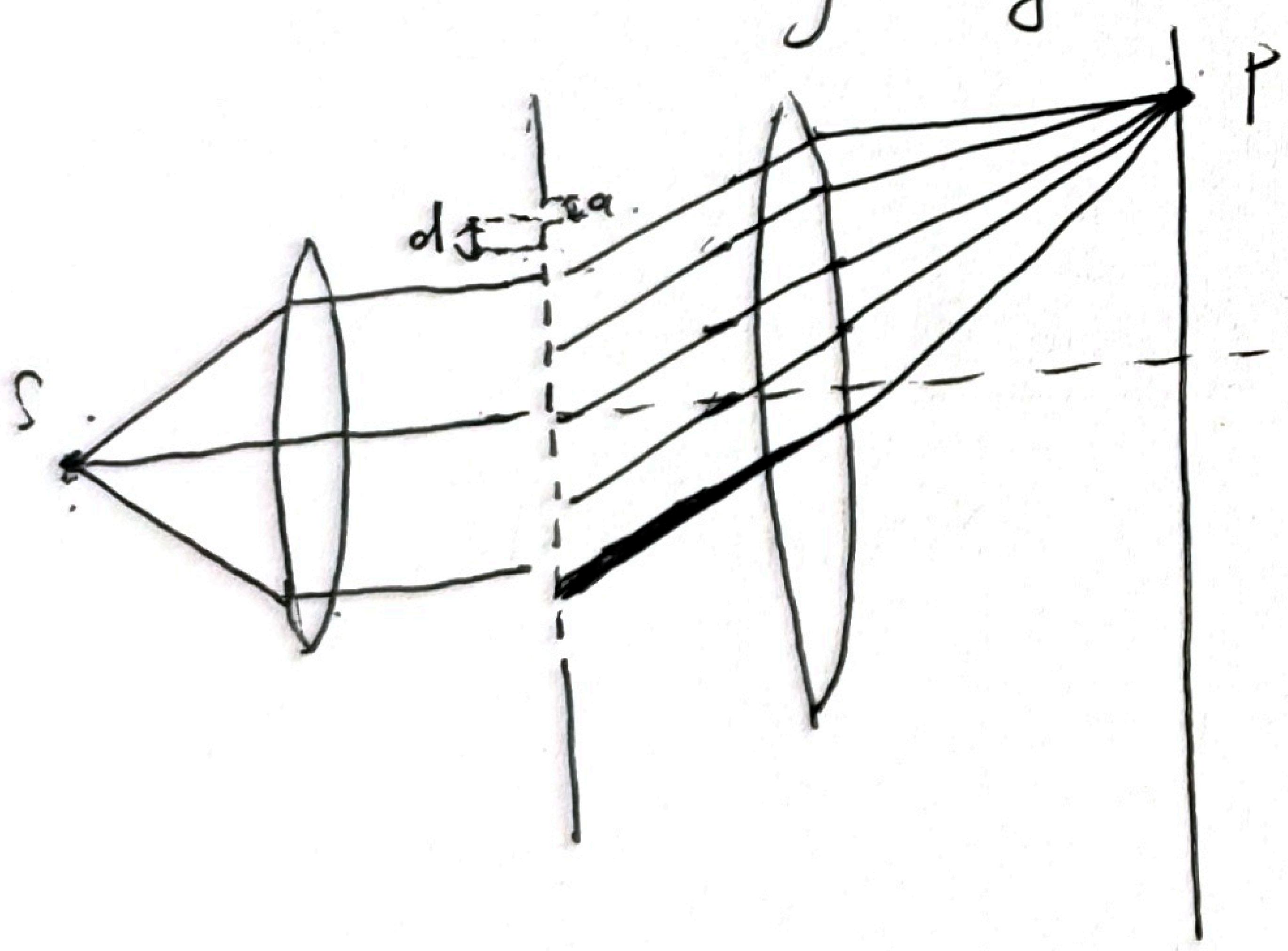
可见，它就是艾里斑的半角宽度，$\theta_{min}=\theta=1.22\frac{\lambda}{D}$.

所以，增加分辨率的方法只有提高望远镜直径.

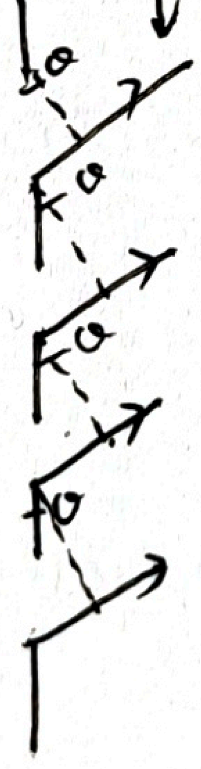
为了方便，定义.

def. resolving power of lens: $R=\frac{1}{\theta_{min}}=\frac{D}{1.22\lambda}$

<The diffraction grating>.



光来的~~光~~已经经过衍射叠加的光来.



$\rightarrow$

$E=2r\sin\frac{N\gamma}{2}$

$E_0'=2r\sin\frac{\gamma}{2}$

$$E=E_0'\frac{\sin\frac{N\gamma}{2}}{\sin\frac{\gamma}{2}}=\frac{\sin\frac{N\gamma}{2}}{\sin\frac{\gamma}{2}}\cdot\frac{\sin\frac{\beta}{2}}{\frac{\beta}{2}}\cdot E_0$$

$$I=I_0\underbrace{\left(\frac{\sin\frac{\beta}{2}}{\frac{\beta}{2}}\right)^2}_{\text{衍射因子}}\underbrace{\left(\frac{\sin\frac{N\gamma}{2}}{\sin\frac{\gamma}{2}}\right)^2}_{\text{干涉因子}},\quad \begin{cases}\beta=\frac{2\pi}{\lambda}a\sin\theta\\ \gamma=\frac{2\pi}{\lambda}d\sin\theta\end{cases}$$

最主要的是干涉因子起作用

光栅方程，因此d仅为缝间的光栅常数

① 主极大 principal maxima. $\begin{cases}\sin\frac{N\gamma}{2}=0\\ \sin\frac{\gamma}{2}=0\end{cases} \Rightarrow \gamma=2k\pi \Rightarrow d\sin\theta=k\lambda$ 或 $\sin\theta=k\frac{\lambda}{d}$

由于 $|\sin\theta|<1$，$k<\frac{d}{\lambda}$，可以判断主极大的个数.

② 零点. $\begin{cases}\sin\frac{N\gamma}{2}=0\\ \sin\frac{\gamma}{2}\neq0\end{cases} \Rightarrow (k+\frac{m}{N})\frac{\lambda}{d}$，$k=0,\pm1,\cdots$，$m=1\cdots N-1$.

所以，每两个主极大之间有 $(N-1)$ 个零点，那么则有 $(N-2)$ 个次极大。

③ 半角宽度（主极大）Half-width of the principal maximum.

$$\left.\begin{array}{ll} d\sin\theta_m = m\lambda & (\text{主极大}) \\ d\sin(\theta_m+\Delta\theta_m) = \left(m+\frac{1}{N}\right)\lambda & (\text{下一极小}) \end{array}\right\} \Rightarrow \boxed{\Delta\theta_m = \frac{\lambda}{Nd\cos\theta_m}}$$

当 N 非常大时 零点非常多，最重要的是主极大和半角宽度。

(4) 衍射因子的作用 $\left(\frac{\sin\frac{\beta}{2}}{\frac{\beta}{2}}\right)^2$ 为包络函数。

可能不同的零点 $a\sin\theta=k\lambda$，$\sin\theta=\frac{k\lambda}{a}$.

这时联想到干涉因子确定的主极大位置 $\sin\theta=\frac{k\lambda}{d}$.

若二者重合，$\sin\theta=\frac{m\lambda}{a}=\frac{n\lambda}{d}$，$\frac{d}{a}=\frac{n}{m}$，

那 如果 d 和 a 之比为有两个整数之比，就会有主极大消失。

这就是 missing order 现象。

<Grating spectrometers>.

角色散本领 dispersion $D=\frac{\delta\theta}{\delta\lambda} \xrightarrow{d\sin\theta=k\lambda} D=\frac{k}{d\cos\theta}$

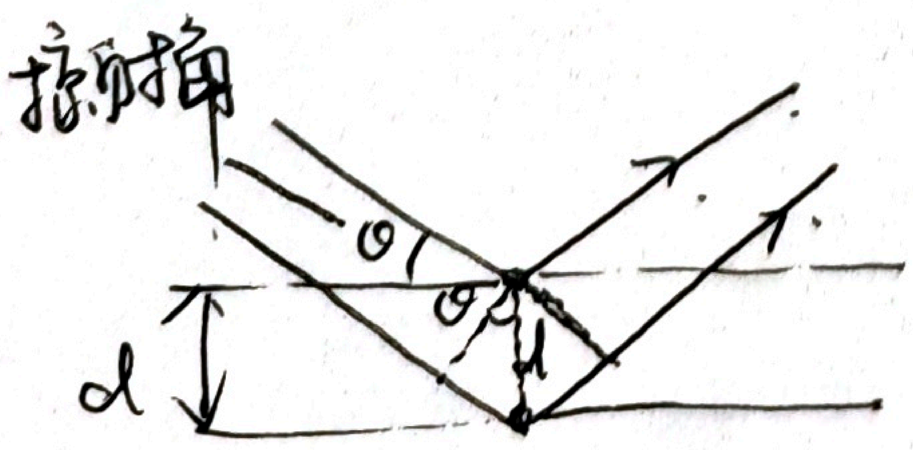
↓ 考虑最小分辨角（半角宽度）$\Delta\theta=\frac{\lambda}{Nd\cos\theta}$

最接近可分辨的波长差 $\delta\lambda=\frac{\delta\theta}{D}=\frac{\Delta\theta}{D}=\frac{\lambda}{Nd\cos\theta}\cdot\frac{d\cos\theta}{k}=\frac{\lambda}{kN}$.

↓ 定义 Resolving power

$R=\frac{\lambda}{\delta\lambda}=\lambda\cdot\frac{kN}{\lambda}=kN$. , $R\uparrow$, Power $\uparrow$.

<Diffraction of X-rays by crystals.>
晶体



$\delta=2d\sin\theta=k\lambda$. (Bragg's Law).

C. Polarization of Light.

<phase shifts>. 相位跃变, 半波损失.

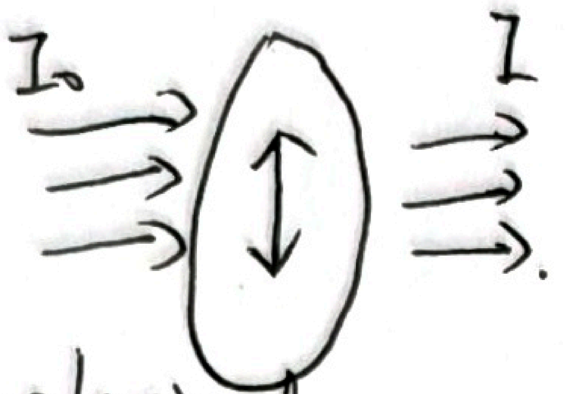
垂直入射或掠入射的情况下,光从光疏介质到光密介质时反射光有半波损失,从光密介质到光疏介质时无半波损失.

任何情况下透射光都没有半波损失.

<States of polarization (SOP)>.

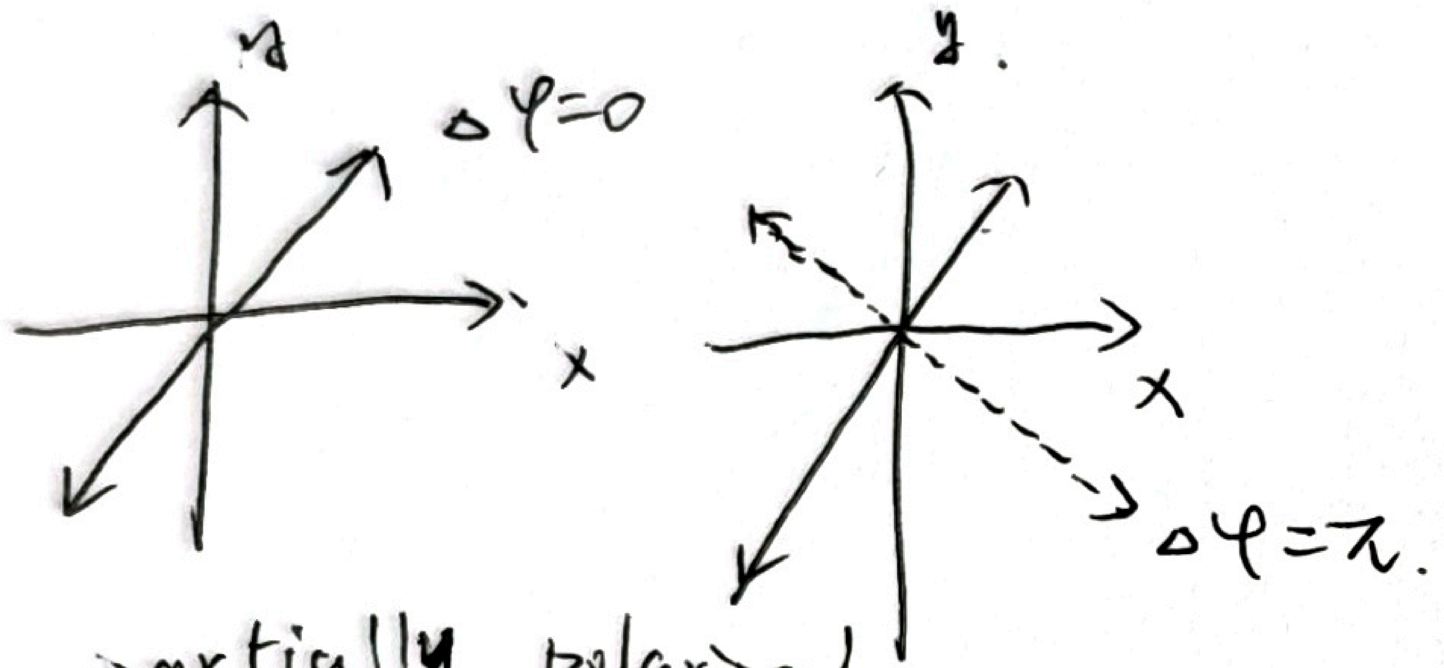
1. unpolarized light or natural light.

自然光

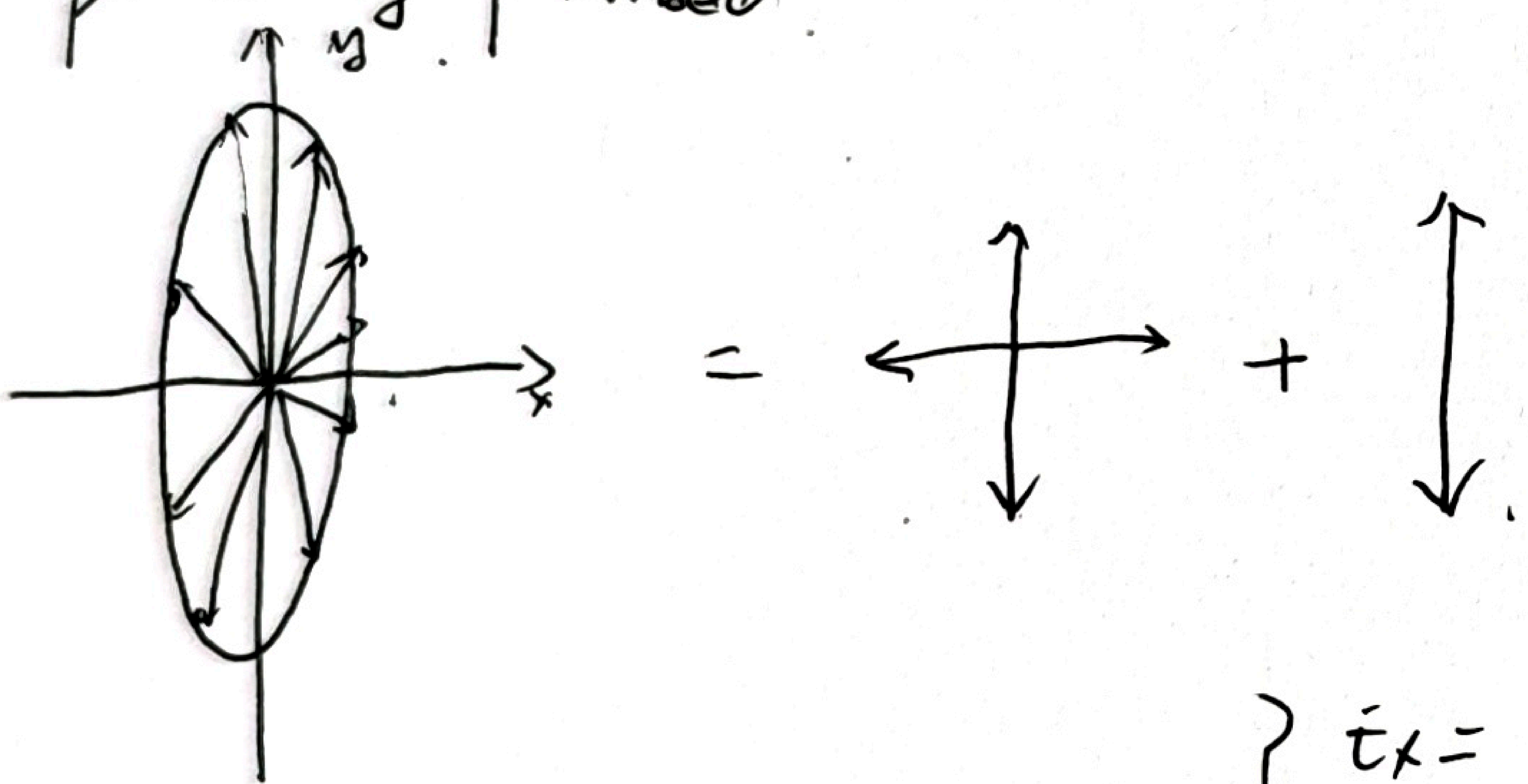


$I = \frac{1}{2} I_0$. (马吕斯定理).

2. linearly polaried.

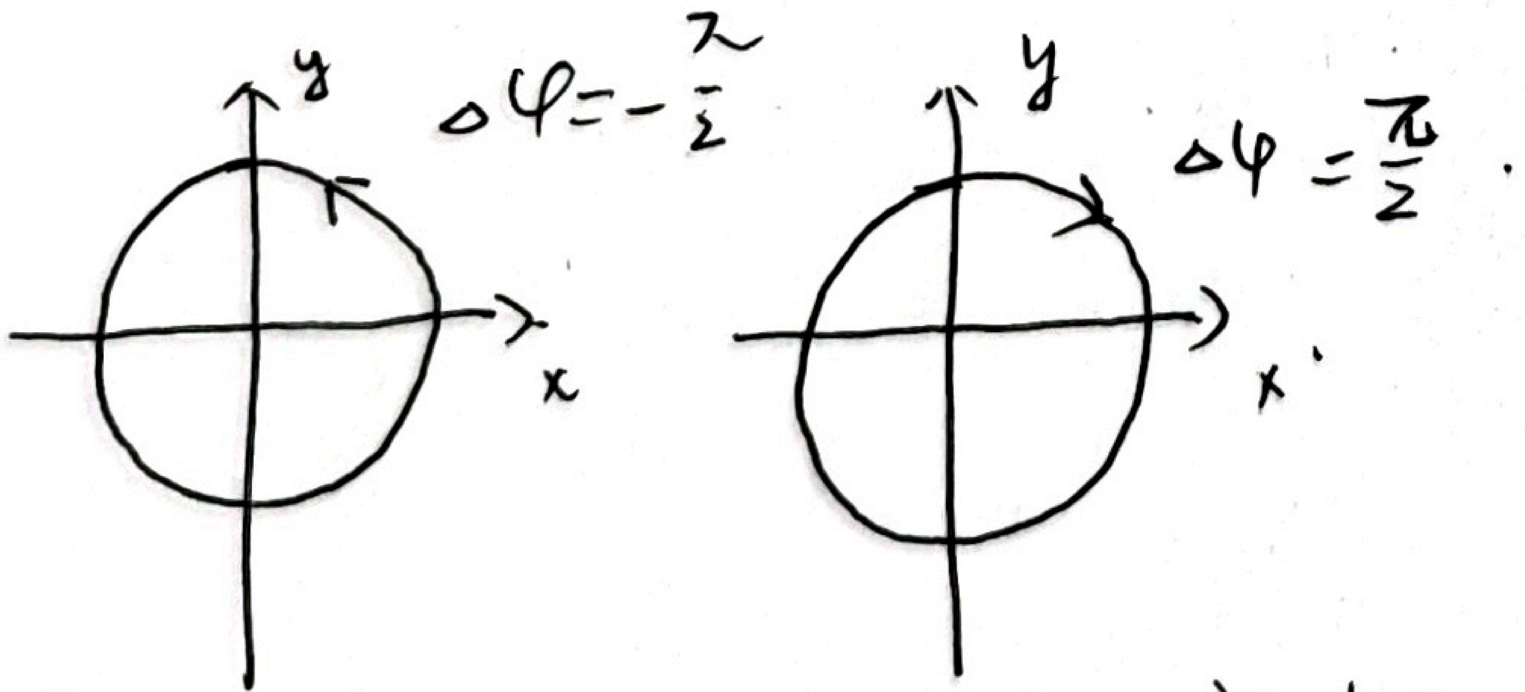


3. partially polarized.



4. Circularly polarized light.

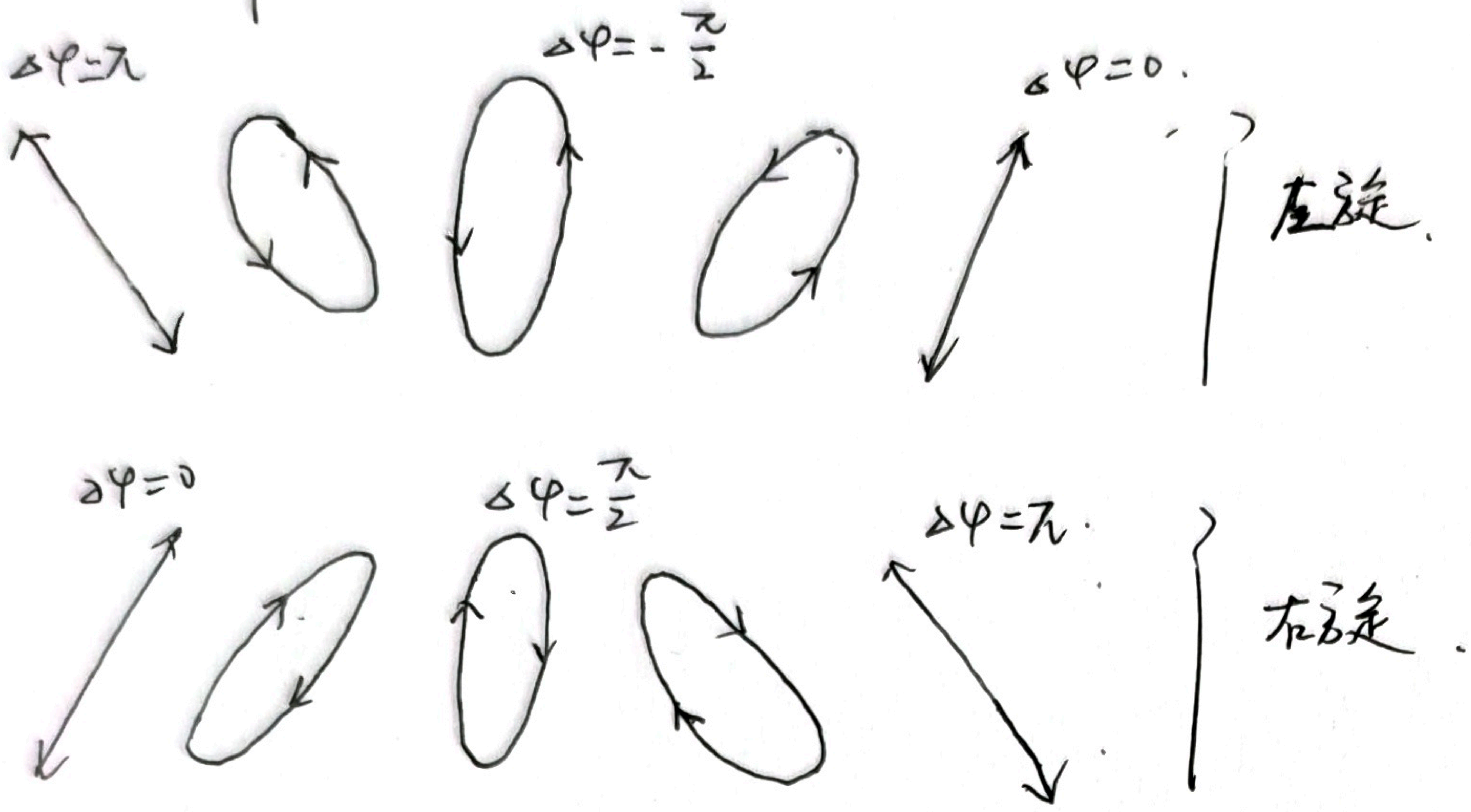
$$\begin{cases} E_x = A\cos\omega t. \\ E_y = A\cos(\omega t + \Delta\varphi). \end{cases}$$



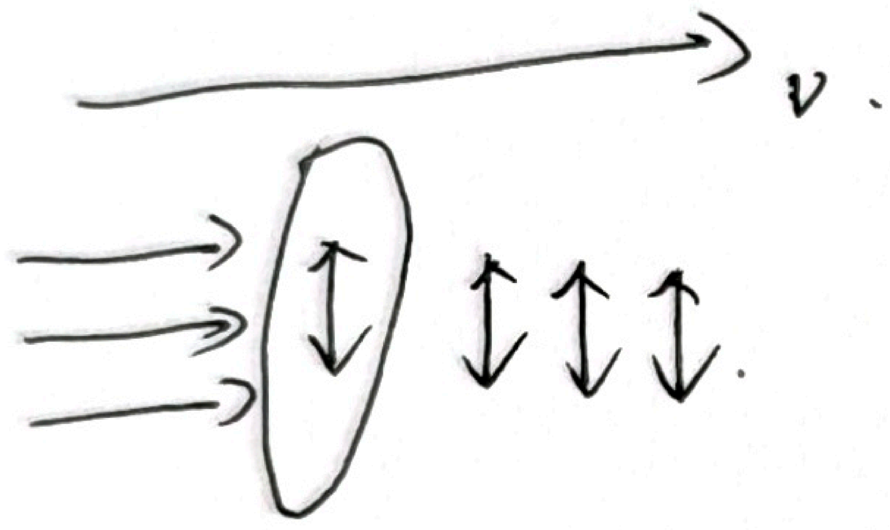
Left-circular light   right-circular light.

5. Elliptically polarized light.

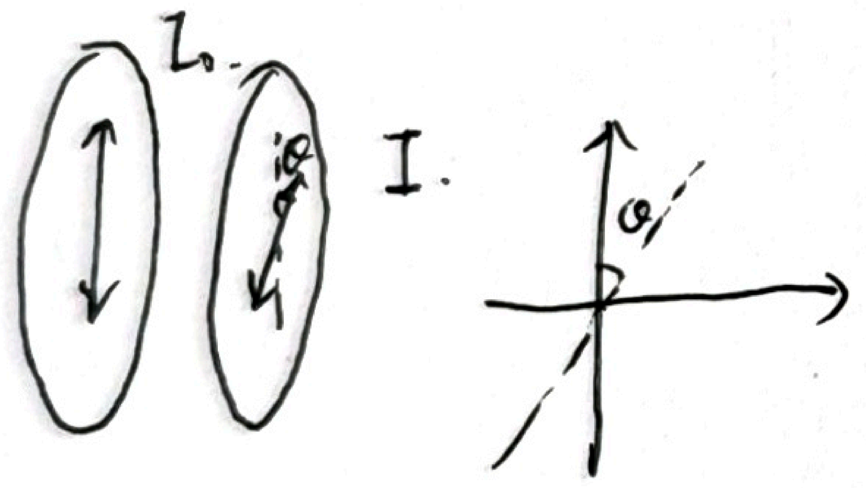
$$\begin{cases} E_x = A_1 \cos \omega t. \\ E_y = A_2 \cos(\omega t + \Delta\varphi). \end{cases}$$



<Polarizer>.



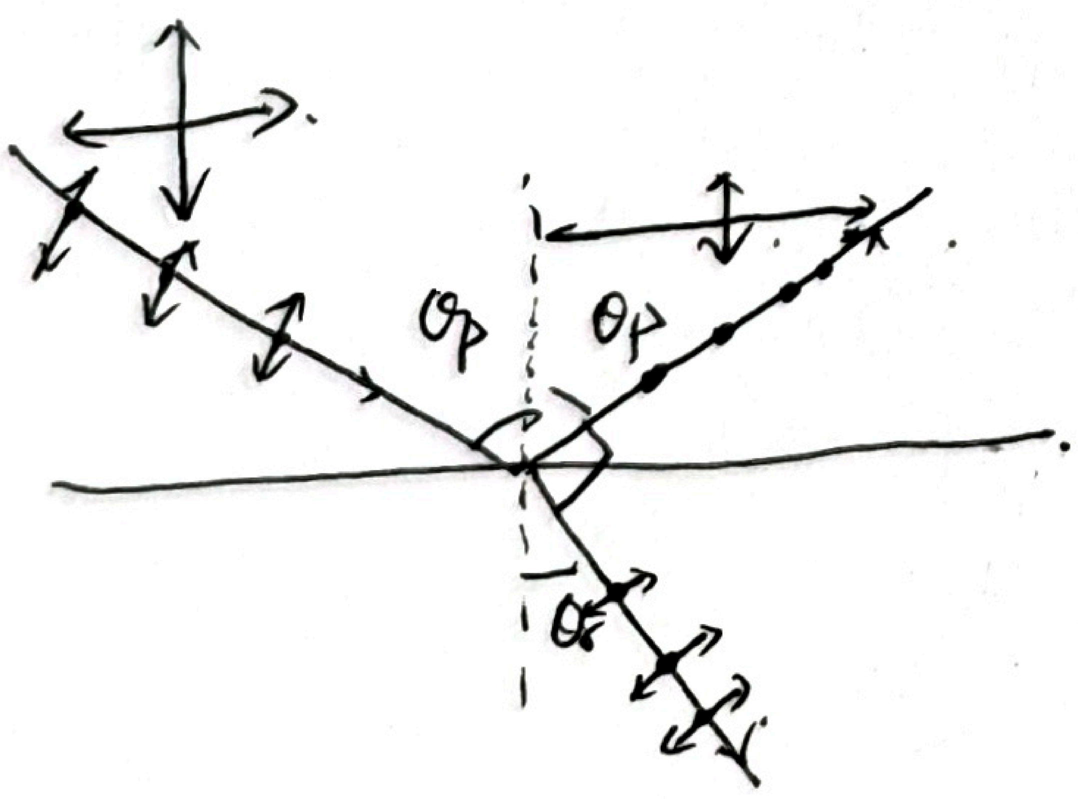
<Malus's law>.



$\boxed{E = E_0 \cos\theta}$.

$\Rightarrow I = I_0 \cos^2\theta$.

<Polarization of reflection and Brewster's angle>.



自然光反射后变为平行于反射面的部分偏振光,

且平行于反射面.

但是，当反射光束和透射光束夹角为90°时.

即 $\begin{cases} \theta_p + \theta_r = 90^\circ \\ n_1 \sin\theta_p = n_2 \sin\theta_r \end{cases} \Rightarrow \tan\theta_p = \frac{n_2}{n_1}$ 时.

the reflected light is totally polarized, its <u>plane of polarization</u> perpendicular to the plane of incidence.

一个方向的反射率为0，另一个方向的反射率不为0，